Parallel Axis Theorem

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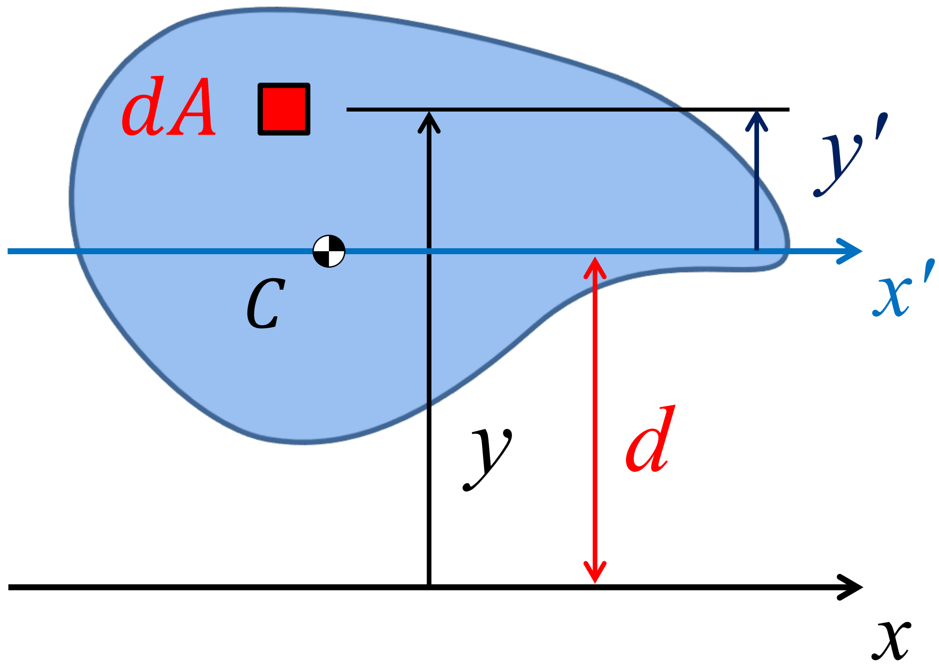
Recall the Definition of the Moment of Inertia of an Area About an Axis

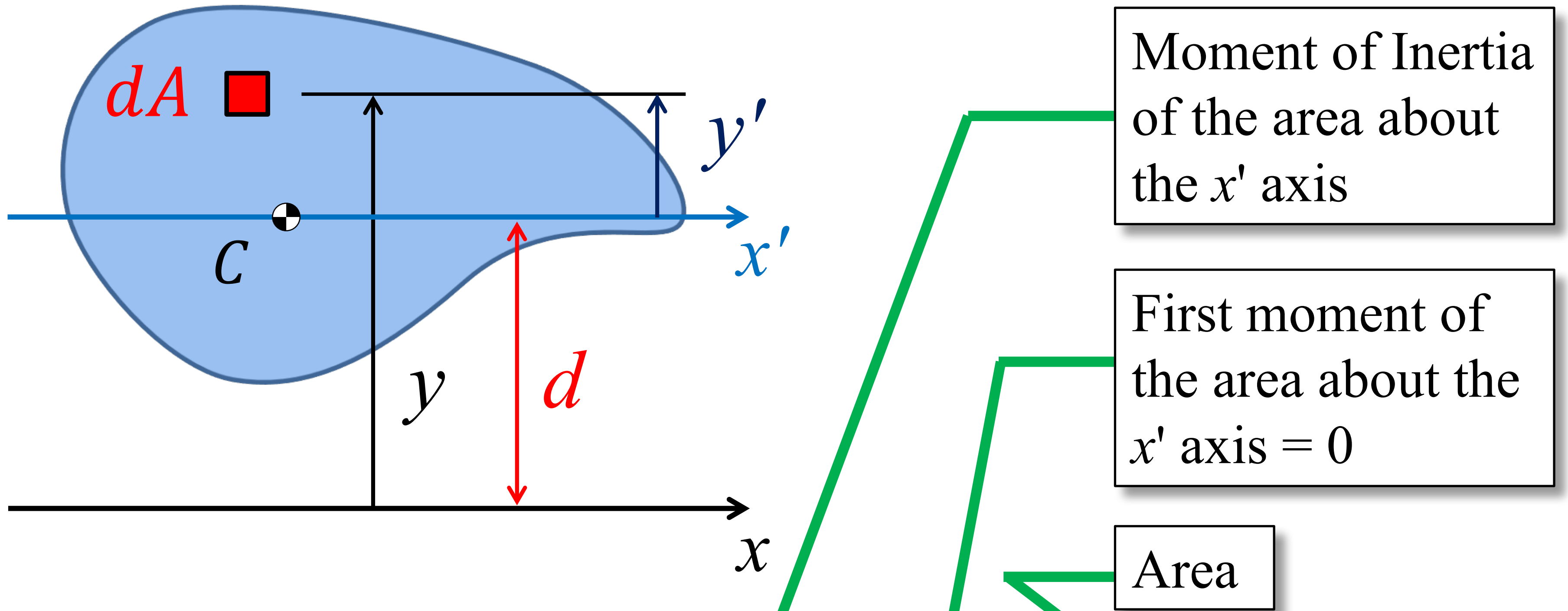


Consider an axis x' that is parallel to the x axis and passes through the centroid of the area. The distance between the two parallel axes is d

$$y = y' + d$$

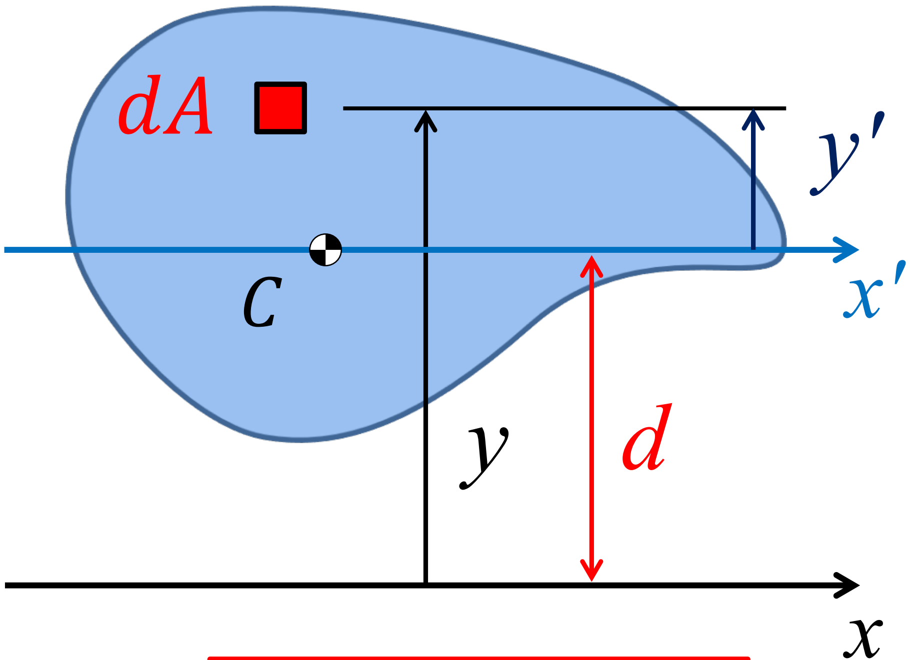
$$I_x = \iint y^2 dA = \iint (y' + d)^2 dA$$

Expand and Examine Terms



$$I_x = \iint (y' + d)^2 dA = \iint y'^2\, dA + 2d \iint y'\, dA + d^2 \iint dA$$

Parallel Axis Theorem



$$I_x = \bar{I}_{x'} + d^2 A$$

General Form

$$I = \bar{I} + d^2 A$$

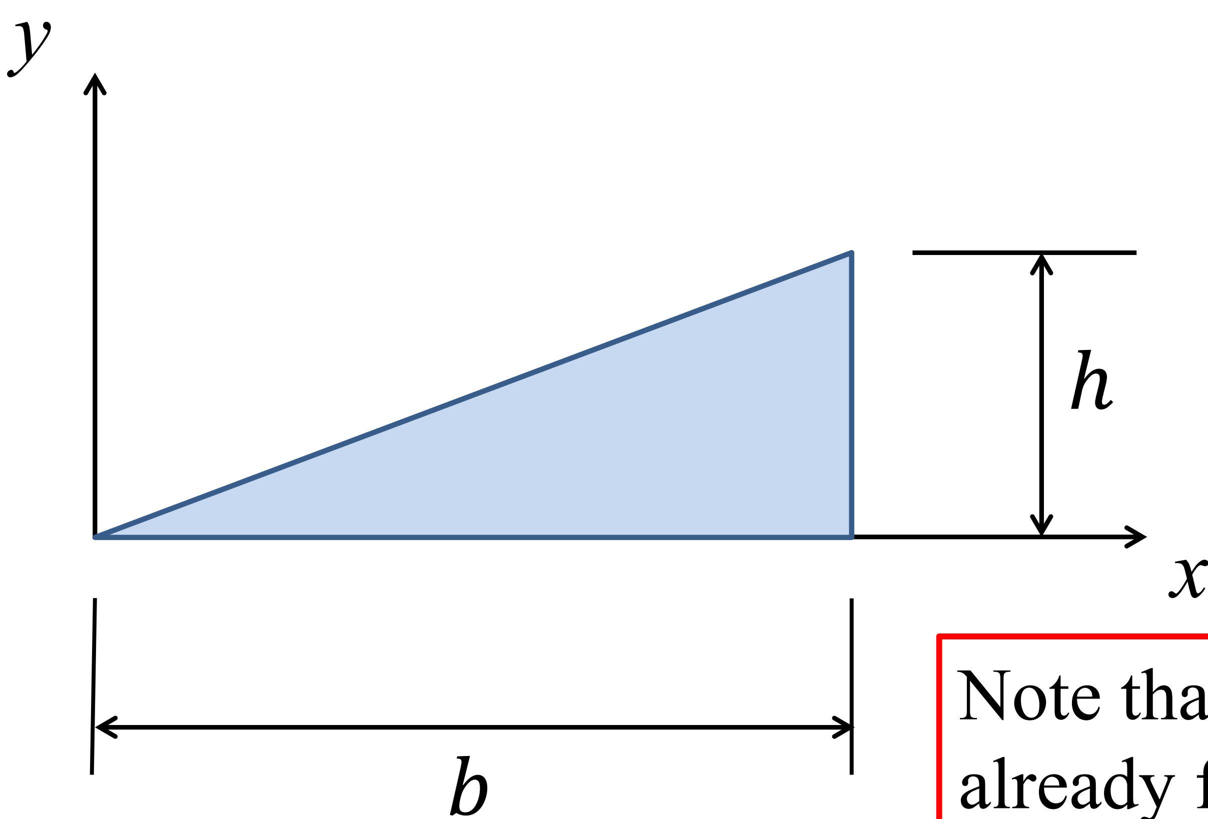
Centroidal Moment of Inertia

$$\bar{I}_{x'} = \iint y'^2 \, dA$$

Parallel Axis Theorem

If we know the moment of inertia of a body about an axis passing through its centroid, we can calculate the body's moment of inertia about any parallel axis

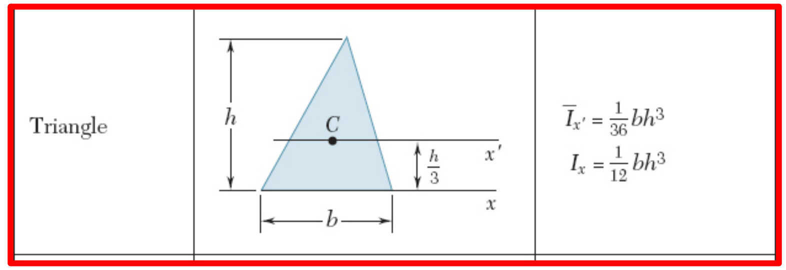
Example Problem

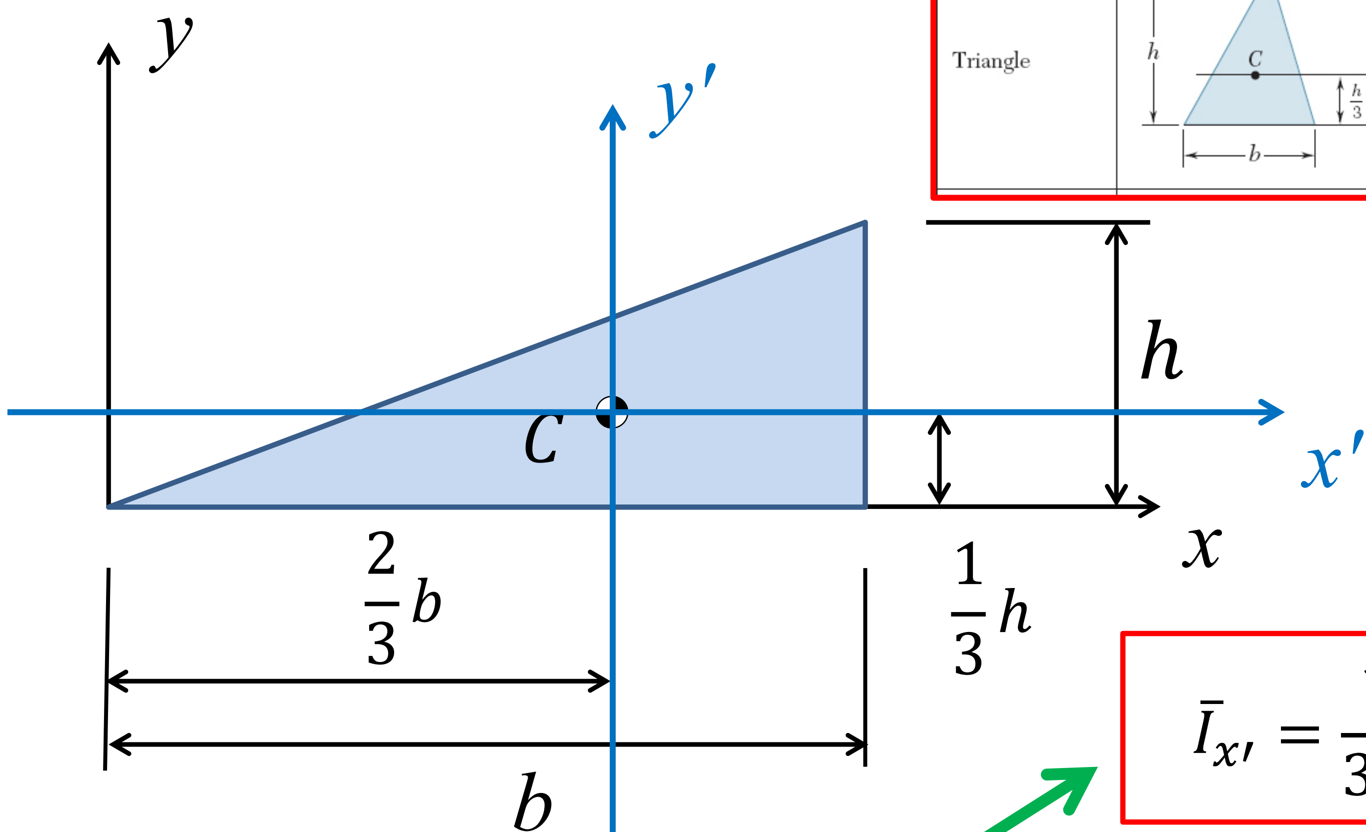


Find the Moment of Inertia of the of the shaded area about the x and y axes shown. Use the Parallel Axis Theorem.

Note that this we have already found I_x , I_y and the location of the centroid for this shape using integration.

Moment of Inertia About Centroidal Axes





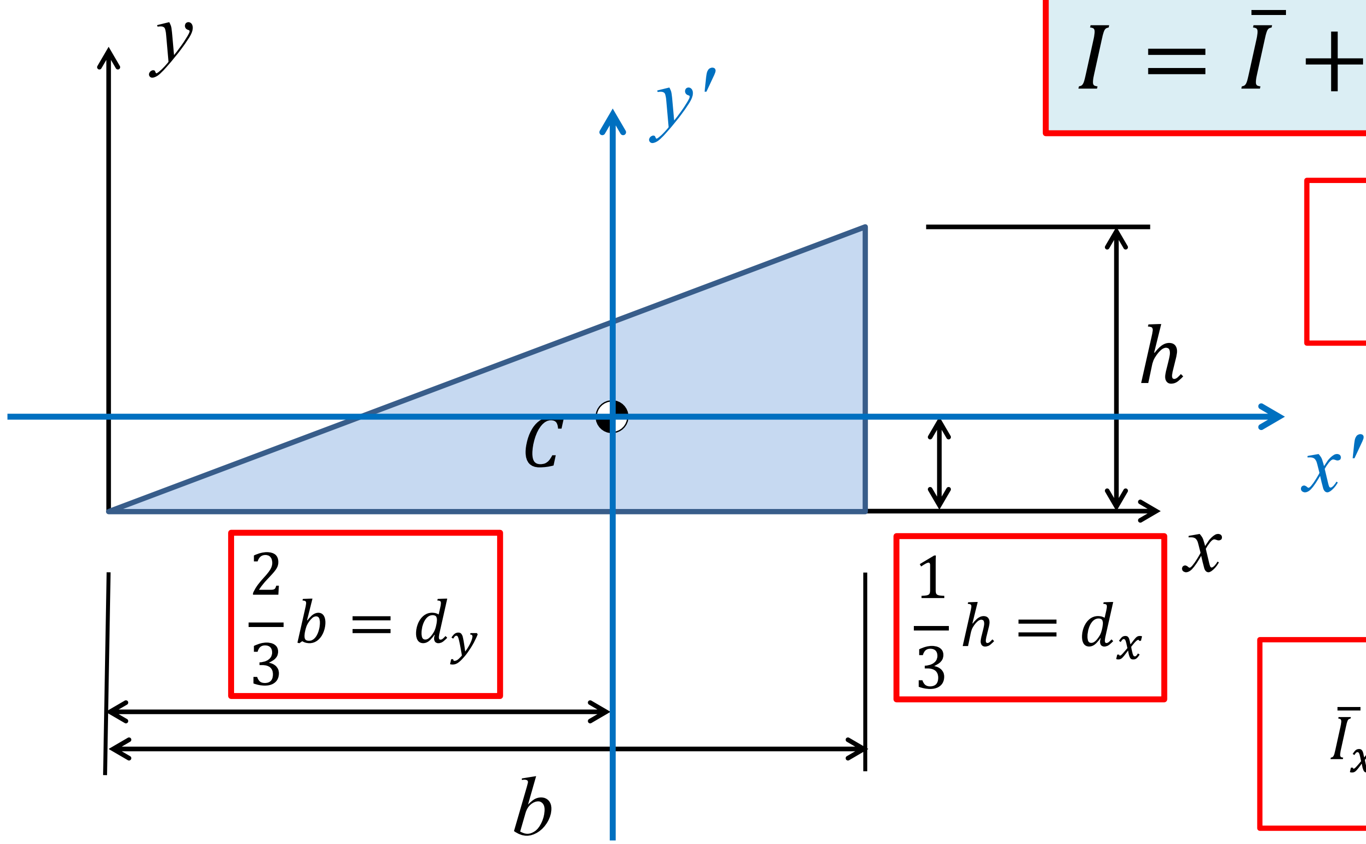
Use Tabulated Solution for $\bar{I}$

$$\bar{I}_{x'} = \frac{1}{36}bh^3$$

$$\bar{I}_{y'} = \frac{1}{36}hb^3$$

Moment of Inertia About Centroidal Axes

$$I = \bar{I} + d^2 A$$



$$\frac{2}{3}b = d_y$$

$$\frac{1}{3}h = d_x$$

$$A = \frac{1}{2}bh$$

$$\bar{I}_{x'} = \frac{1}{36}bh^3$$

$$I_x = \bar{I}_{x'} + {d_x}^2 A = \frac{1}{36}bh^3 + \left(\frac{1}{3}h\right)^2 \frac{1}{2}bh$$

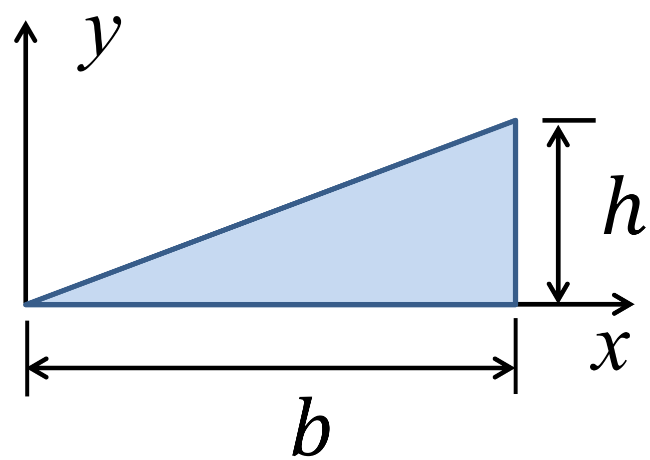
$$\bar{I}_{y'} = \frac{1}{36}hb^3$$

Moment of Inertia About the x Axis

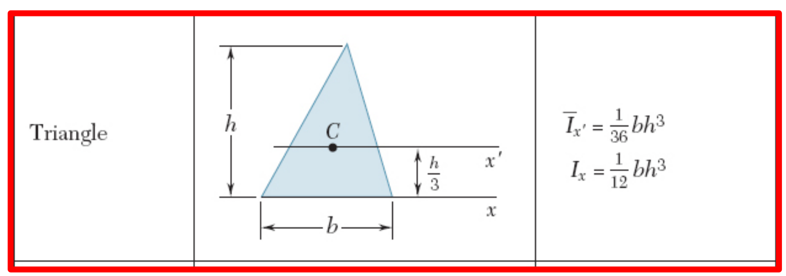
$$I_x = \bar{I}_{x'} + {d_x}^2 A = \frac{1}{36} bh^3 + \left(\frac{1}{3}h\right)^2 \frac{1}{2} bh$$

$$I_x = \frac{1}{36} bh^3 + \frac{1}{18} bh^3$$

$$I_x = \frac{3}{36} bh^3 = \frac{1}{12} bh^3$$



Agrees with both the tabulated solution and our result from integration

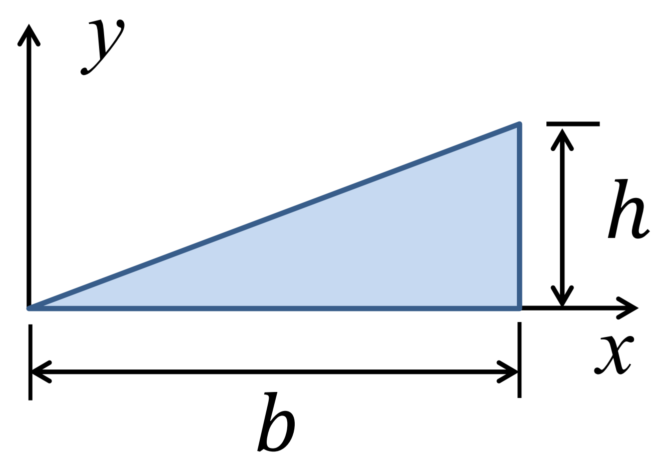


Moment of Inertia About the *y* Axis

$$I_y = \bar{I}_{y'} + d_y{}^2 A = \frac{1}{36} hb^3 + \left(\frac{2}{3} b\right)^2 \frac{1}{2} bh$$

$$I_y = \frac{1}{36} hb^3 + \frac{4}{18} hb^3$$

$$I_y = \frac{9}{36} hb^3 = \frac{1}{4} hb^3$$



Agrees with our result from integration