Statically Indeterminate Frame Example

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Steps in Solving an Indeterminate Structure using the Force Method

Determine degree of Indeterminacy
Let *n* =degree of indeterminacy
(i.e. the structure is indeterminate to the nth degree)

Define Primary Structure and the *n* Redundants

Define the Primary Problem

Solve for the *n* Relevant Deflections in Primary Problem

Define the *n* Redundant Problems

Solve for the *n* Relevant Deflections in each Redundant Problem

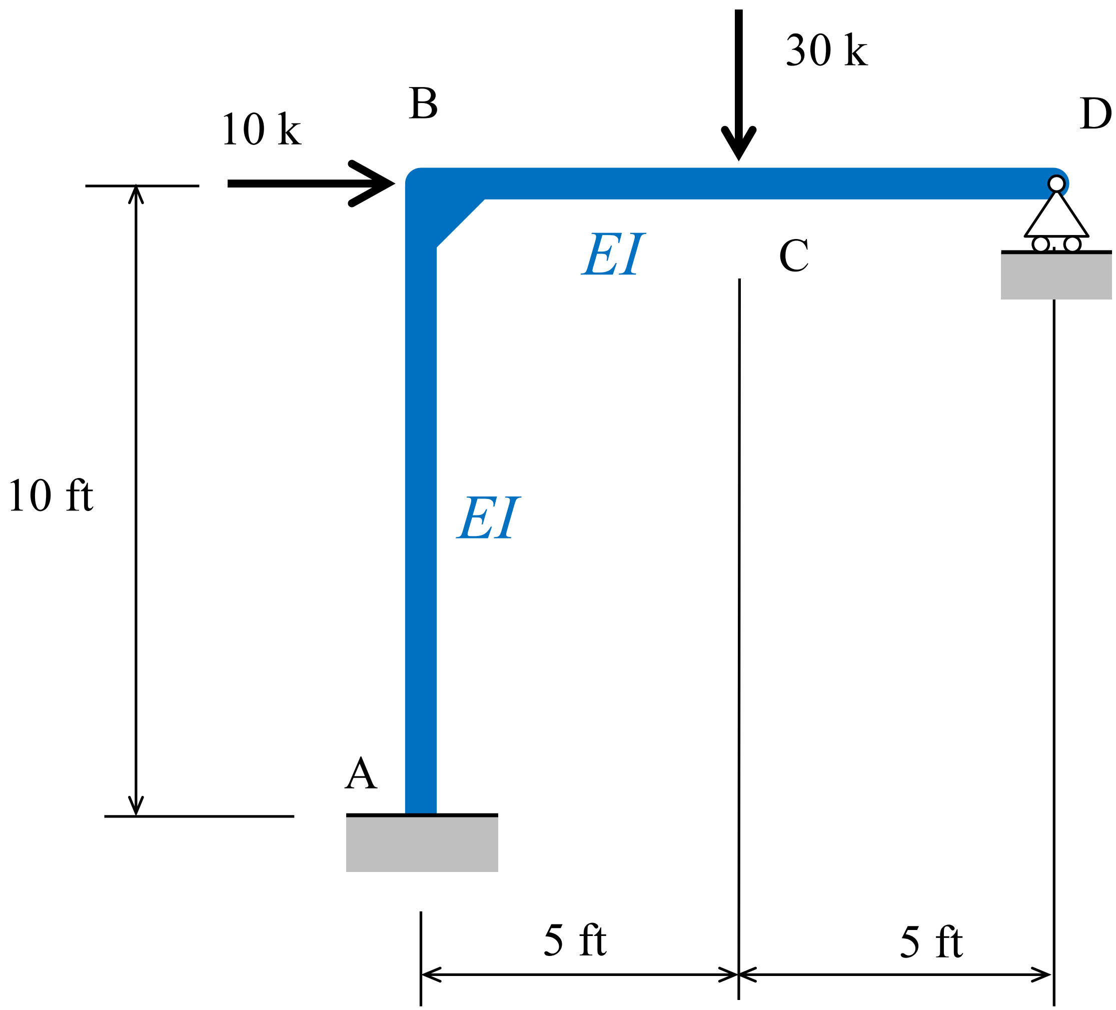
Write the *n* Compatibility Equations at Relevant Points

Solve the *n* Compatibility Equations to find the *n* Redundants

Use the Equations of Equilibrium to solve for the remaining unknowns

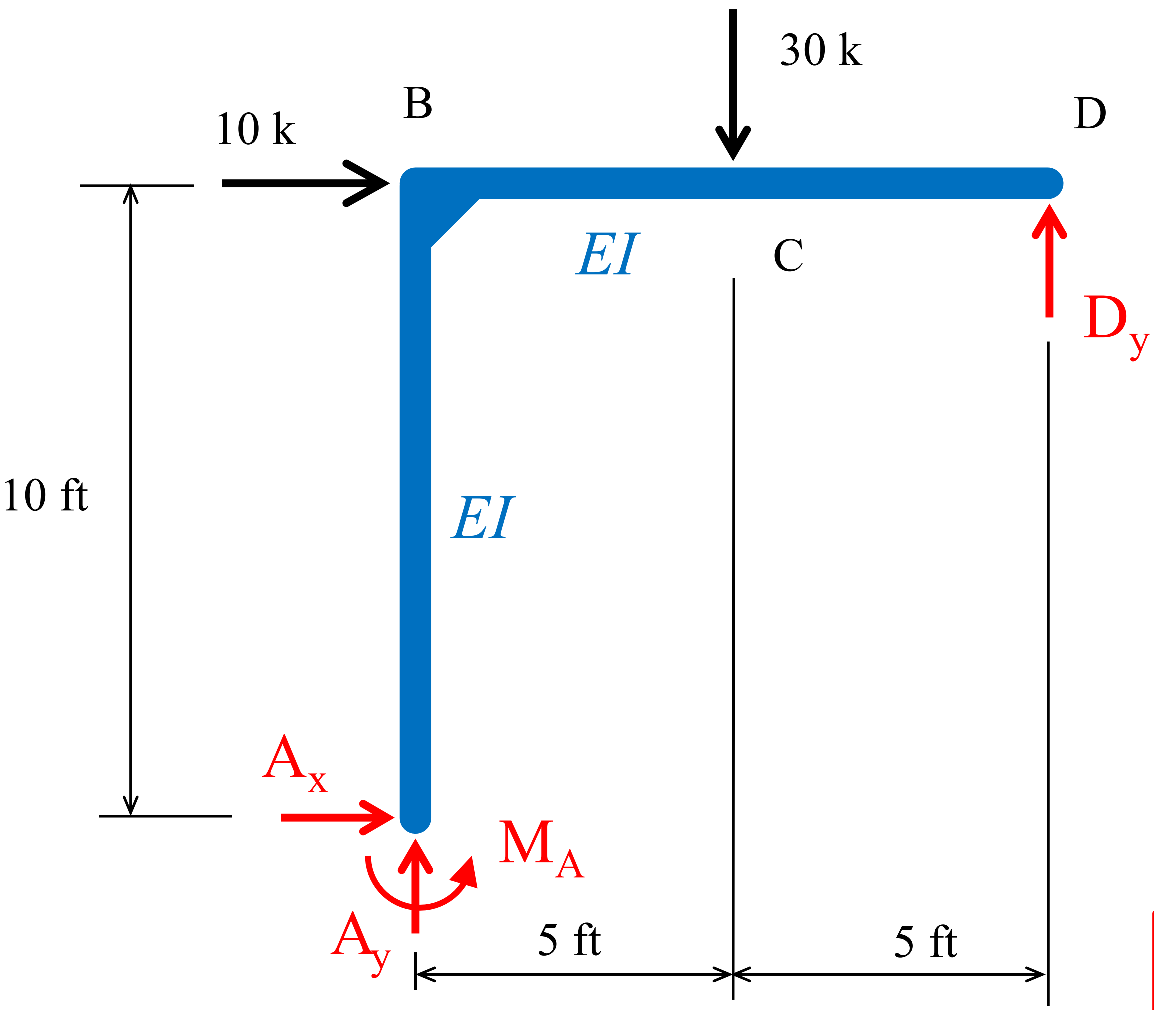
Construct Internal Force Diagrams (if necessary)

Example Problem



For the indeterminate frame subjected to the point loads shown, find the support reactions and draw the bending moment diagram for the frame. *EI* is the same for both the horizontal and vertical members.

FBD of the Frame



Frame is stable

$X = 4$

$3n = 3(1) = 3$

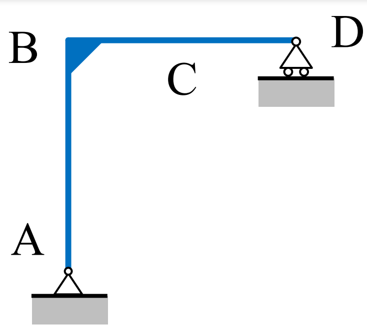
Statically Indeterminate to the 1st degree

Define Primary Structure and Redundant

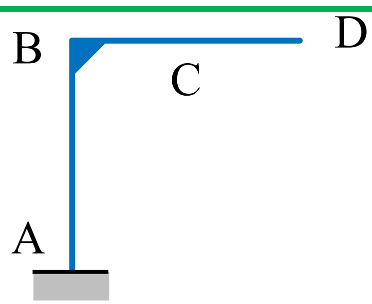
- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.

Primary Structure

Redundant



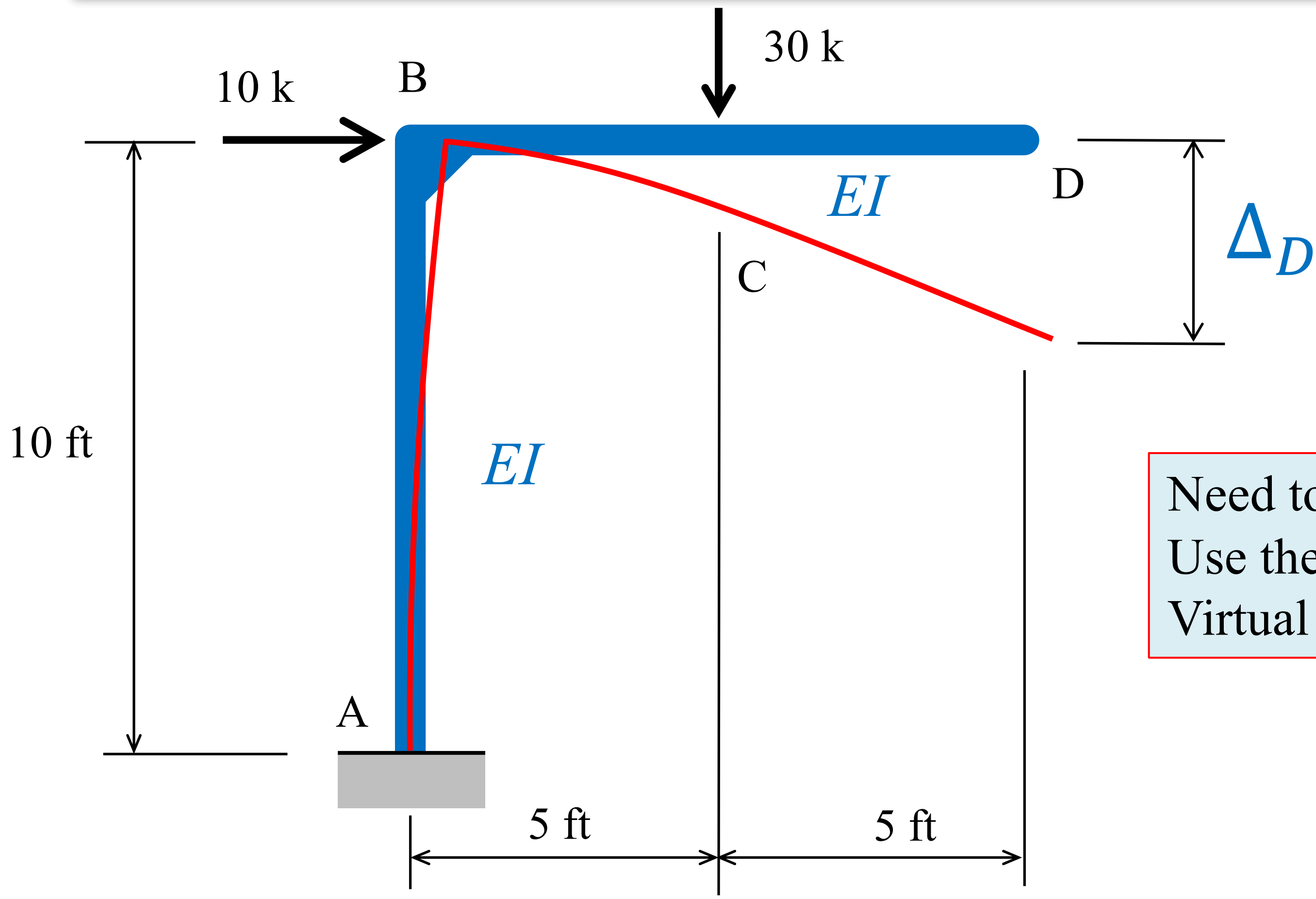
M_A



D_y

Define and Solve the Primary Problem

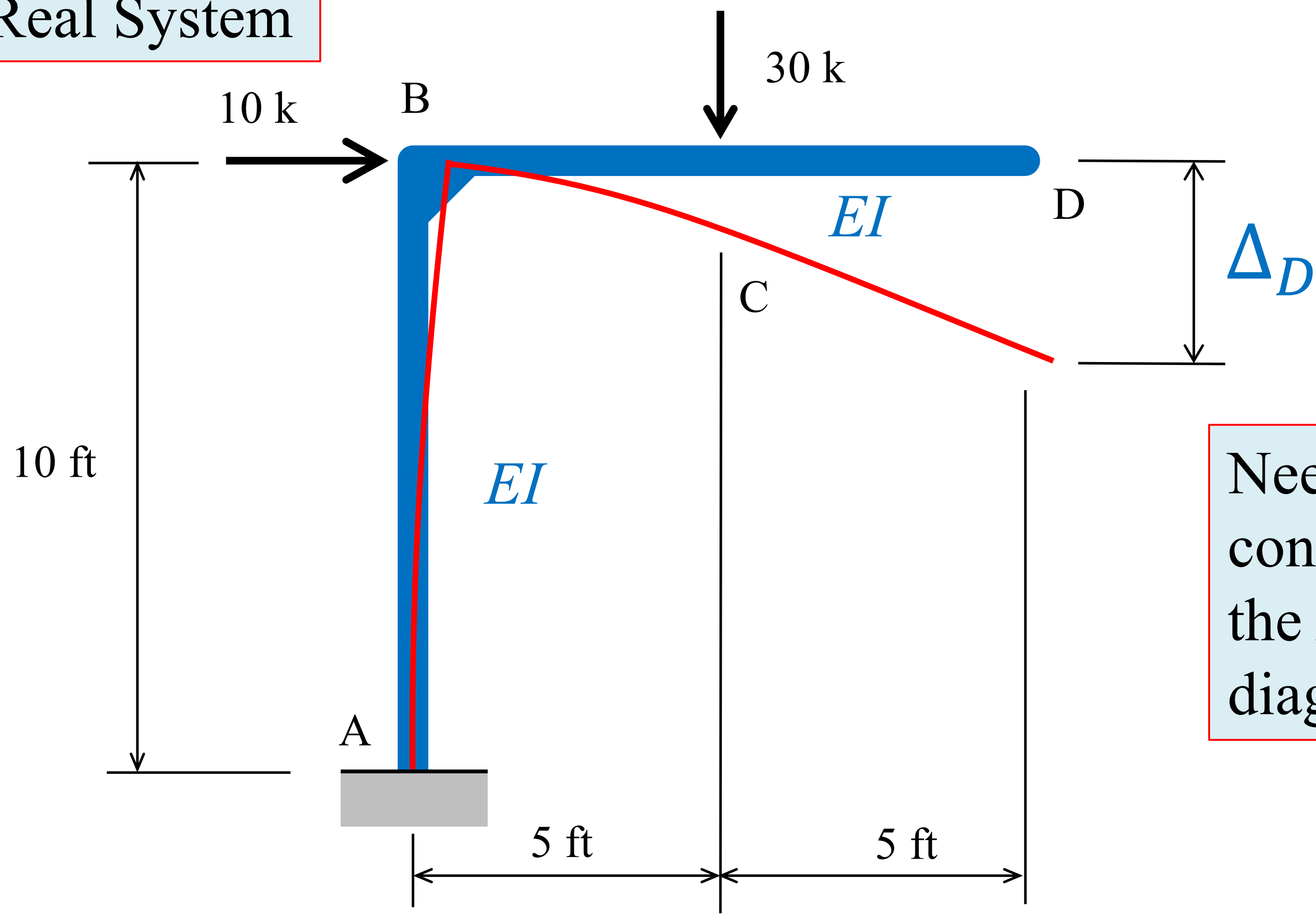
- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.



Need to find Δ_D
Use the Principle of Virtual Work

Solve the Primary Problem

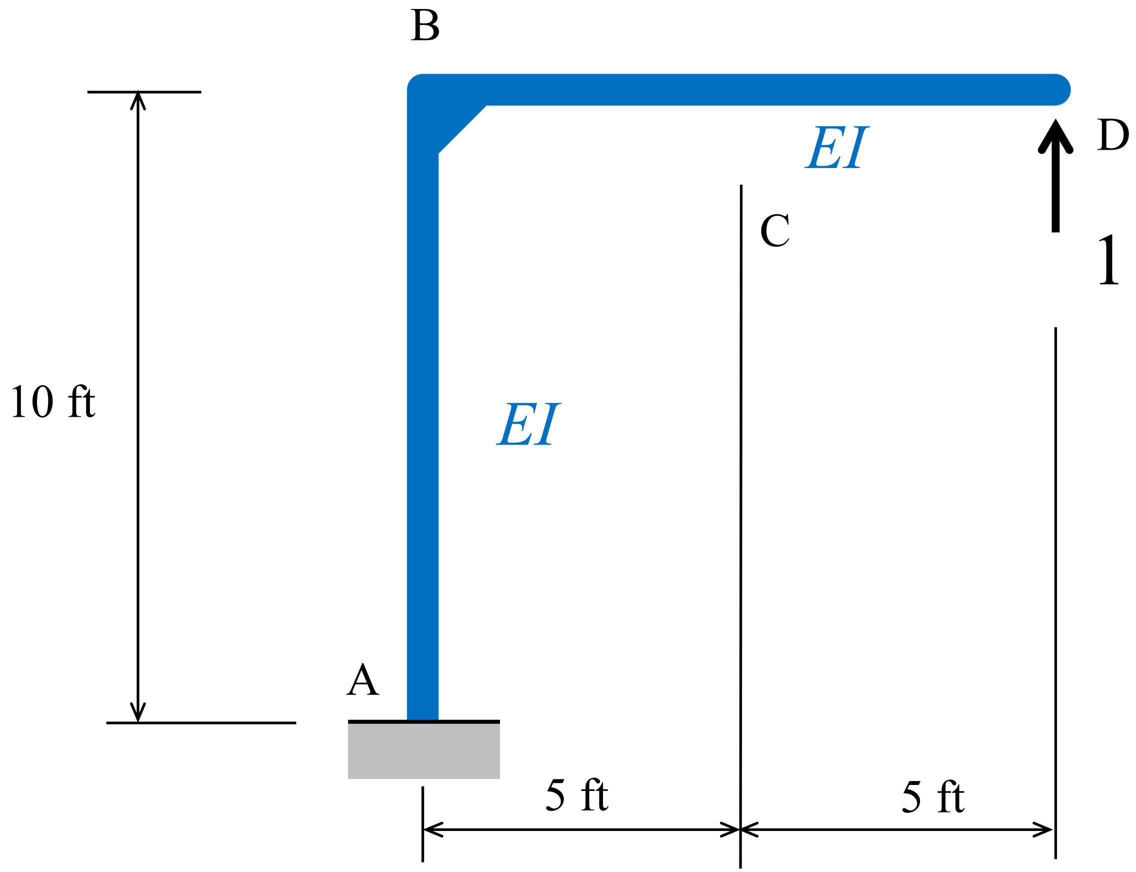
Real System



Need to construct the M_p diagram

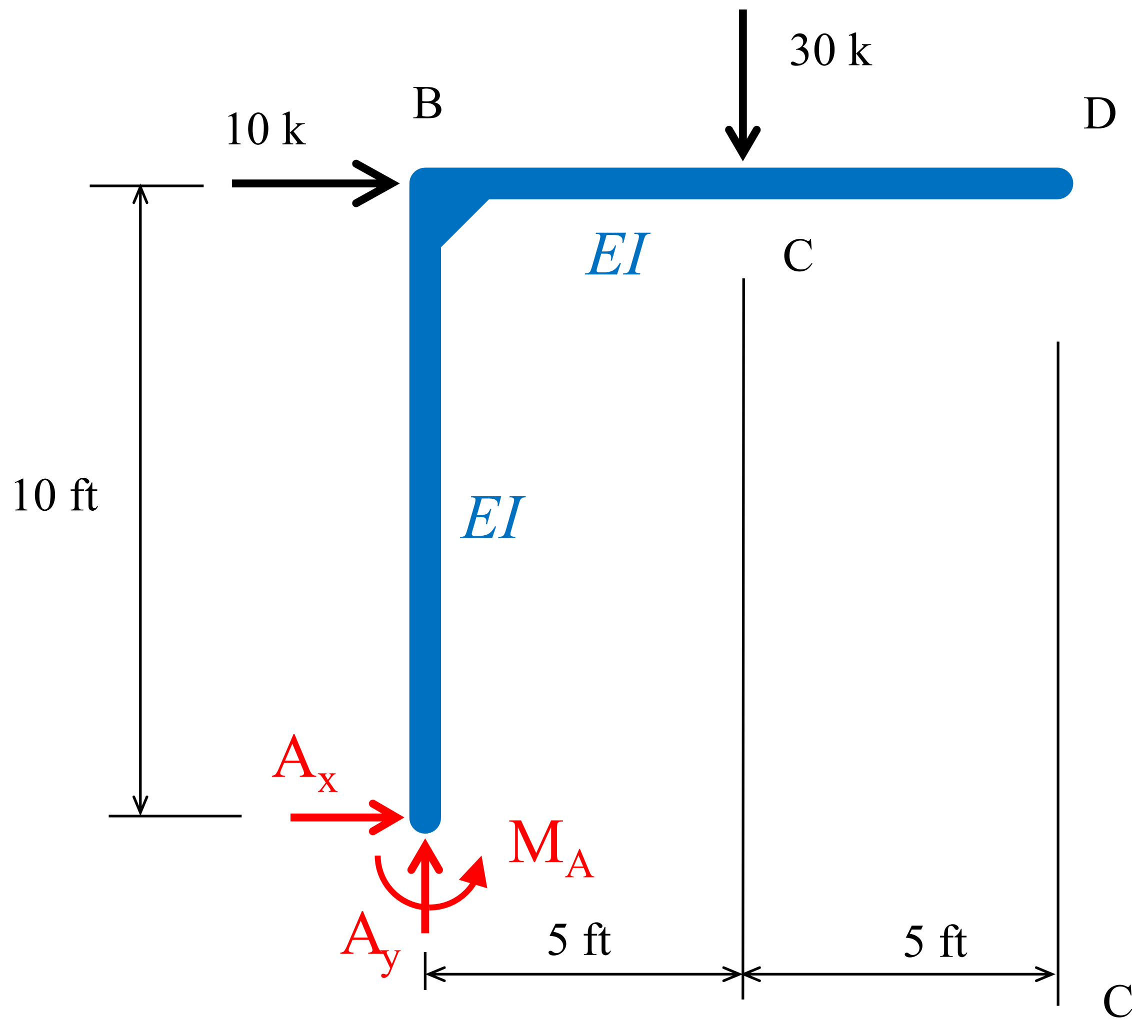
Solve the Primary Problem

Virtual System to measure Δ_D



Need to construct the M_Q diagram

FBD of the Primary Problem



$$\circlearrowleft + \sum M_A = 0$$

$M_A = 250$ k-ft

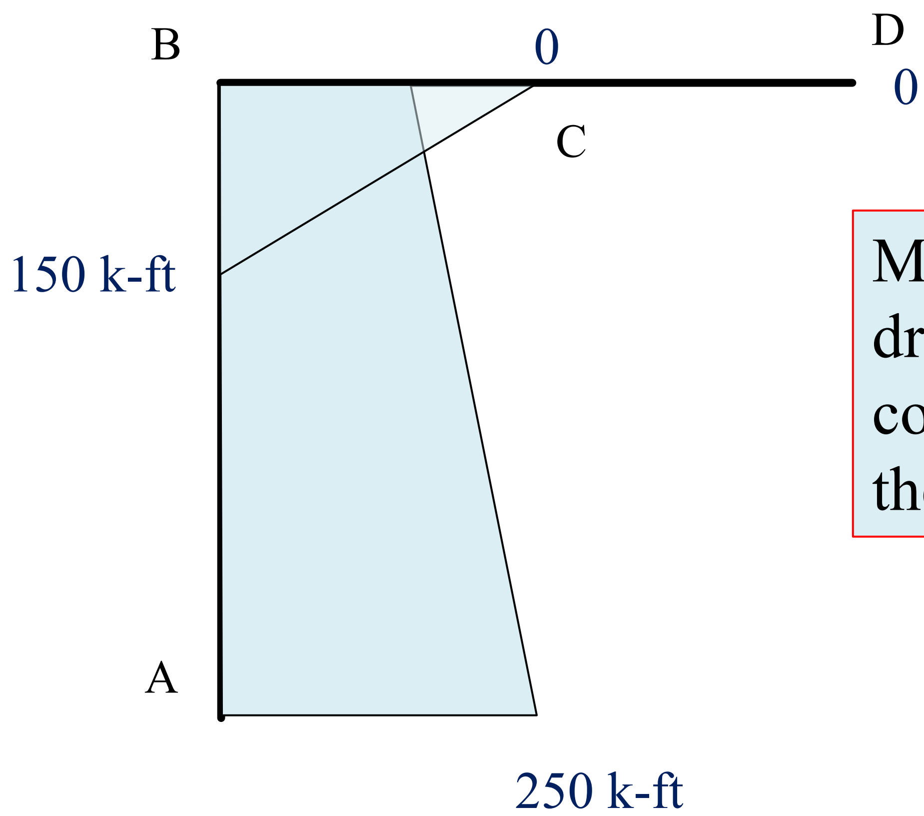
$$\xrightarrow{+} \sum F_x = 0$$

$A_x = -10$ k

$$+\uparrow \sum F_y = 0$$

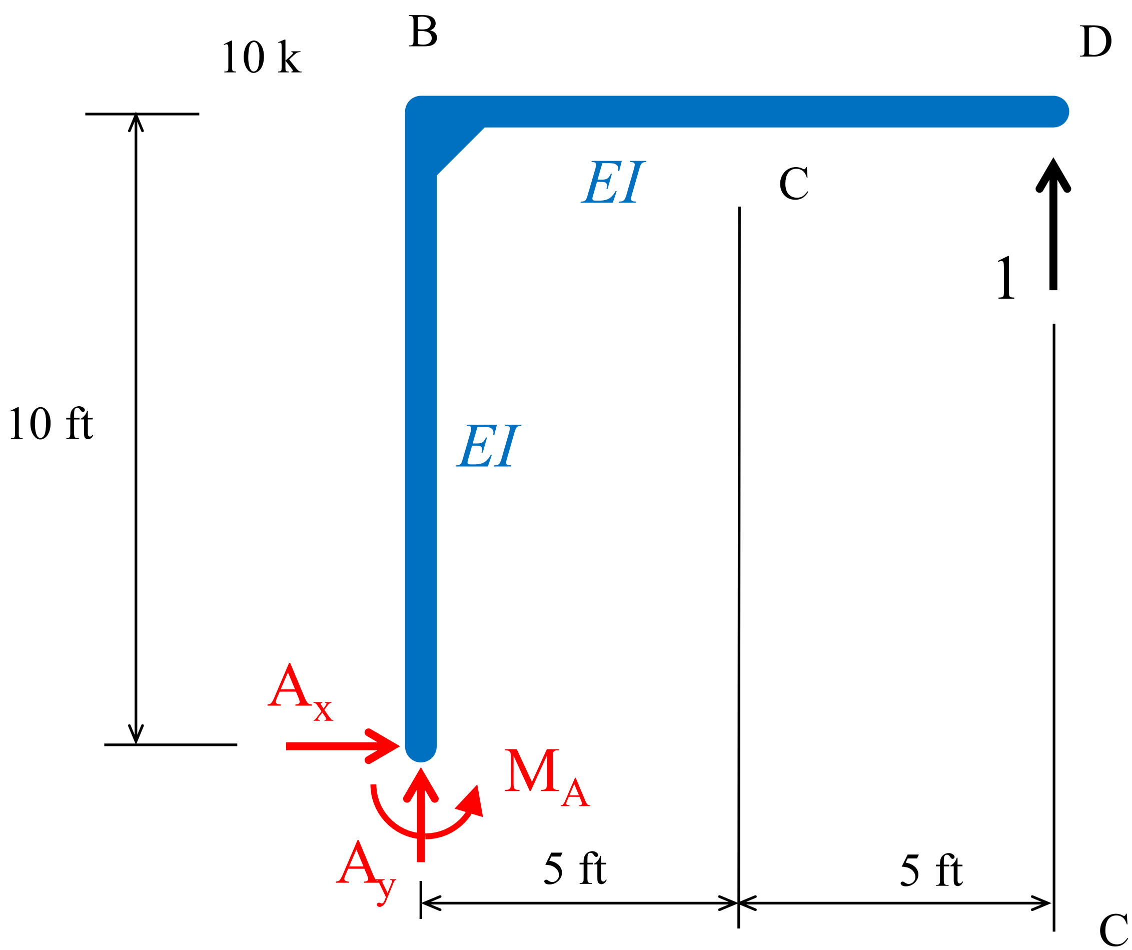
$A_y = 30$ k

M_P Diagram for the Primary Problem



Moment diagram is drawn on the compression side of the member

FBD of the Virtual System



$$\circlearrowleft + \sum M_A = 0$$

$$M_A = 10 \text{ ft}$$

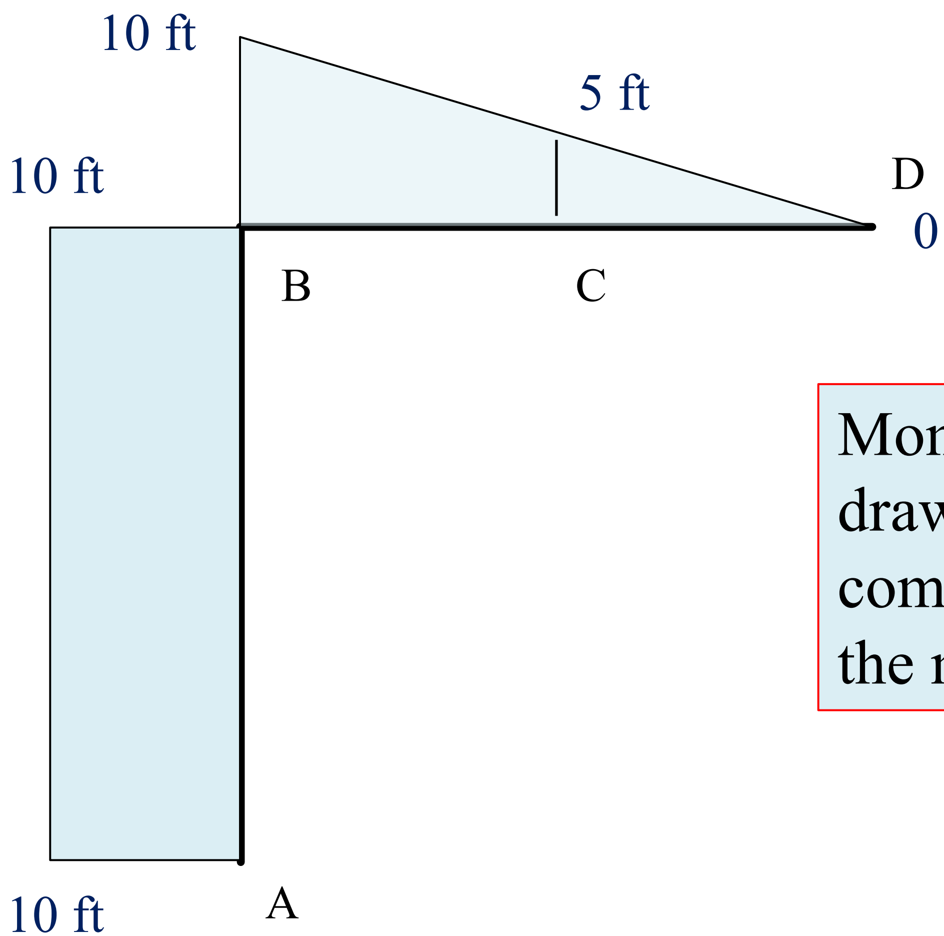
$$\xrightarrow{+} \sum F_x = 0$$

$$A_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$A_y = -1$$

M_Q Diagram for the Primary Problem

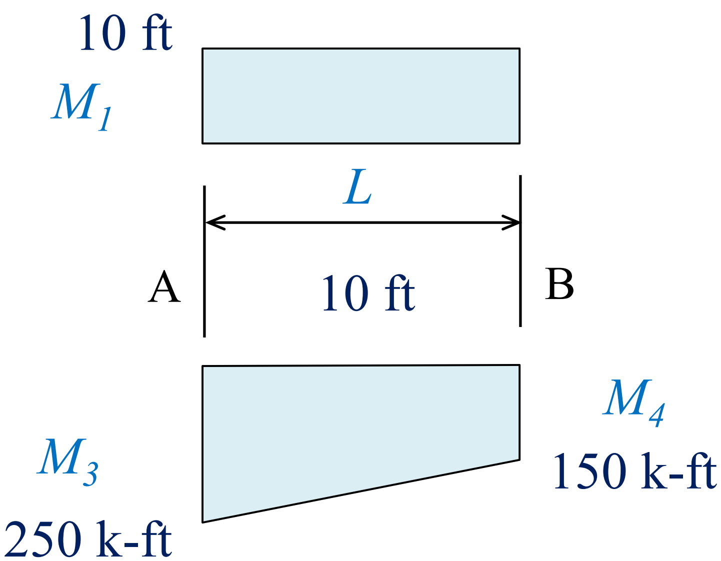


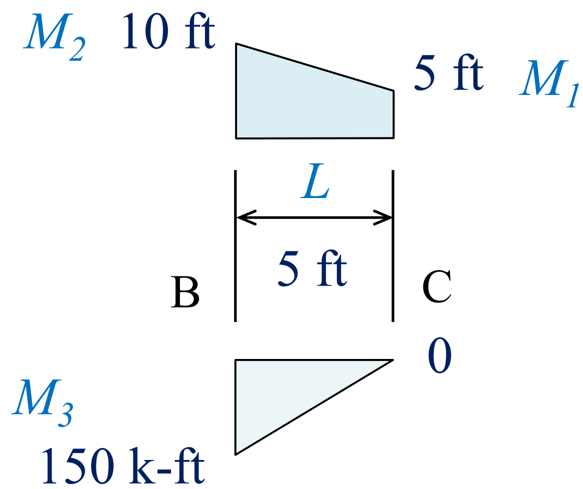
Moment diagram is drawn on the compression side of the member

Solve the Primary Problem

$$1 \cdot \Delta_D = \frac{1}{EI}\int_0^L M_Q M_P \, dx$$

$$\Delta_D = -\frac{23{,}125 \text{ k-ft}^3}{EI}$$





$$-\frac{1}{2}M_1(M_3 + M_4)L$$

$$-\frac{1}{6}(M_1 + 2M_2)M_3L$$

$$-\left(\frac{1}{2}\right)(10 \text{ ft})(250 \text{ k-ft} + 150 \text{ k-ft})(10 \text{ ft})$$

$-20{,}000$ k-ft^3

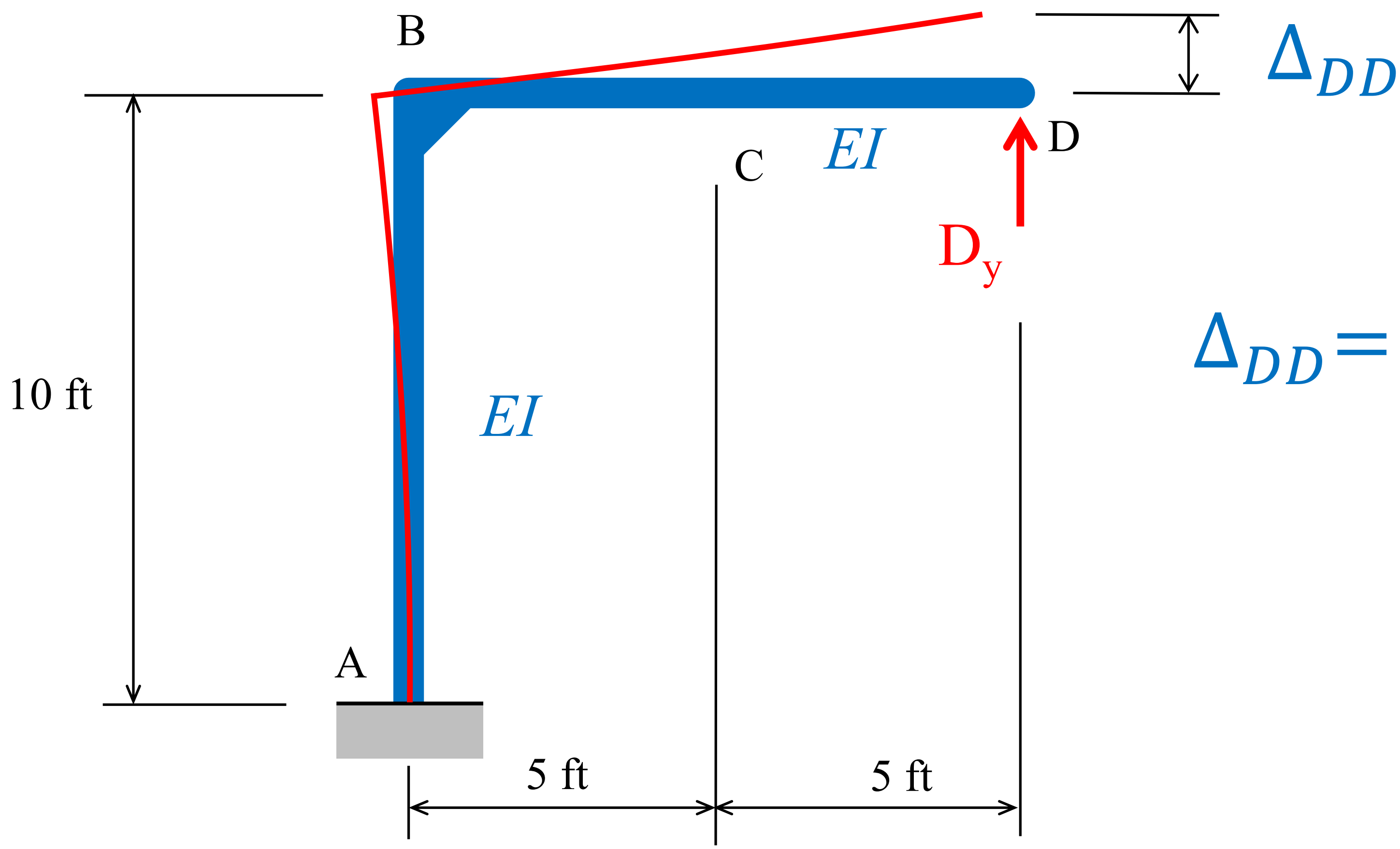
$$-\left(\frac{1}{6}\right)[5 \text{ ft} + 2(10 \text{ ft})](150 \text{ k-ft})(5 \text{ ft})$$

$-3{,}125$ k-ft^3

redundant for each redundant problem.

- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

Redundant Problem

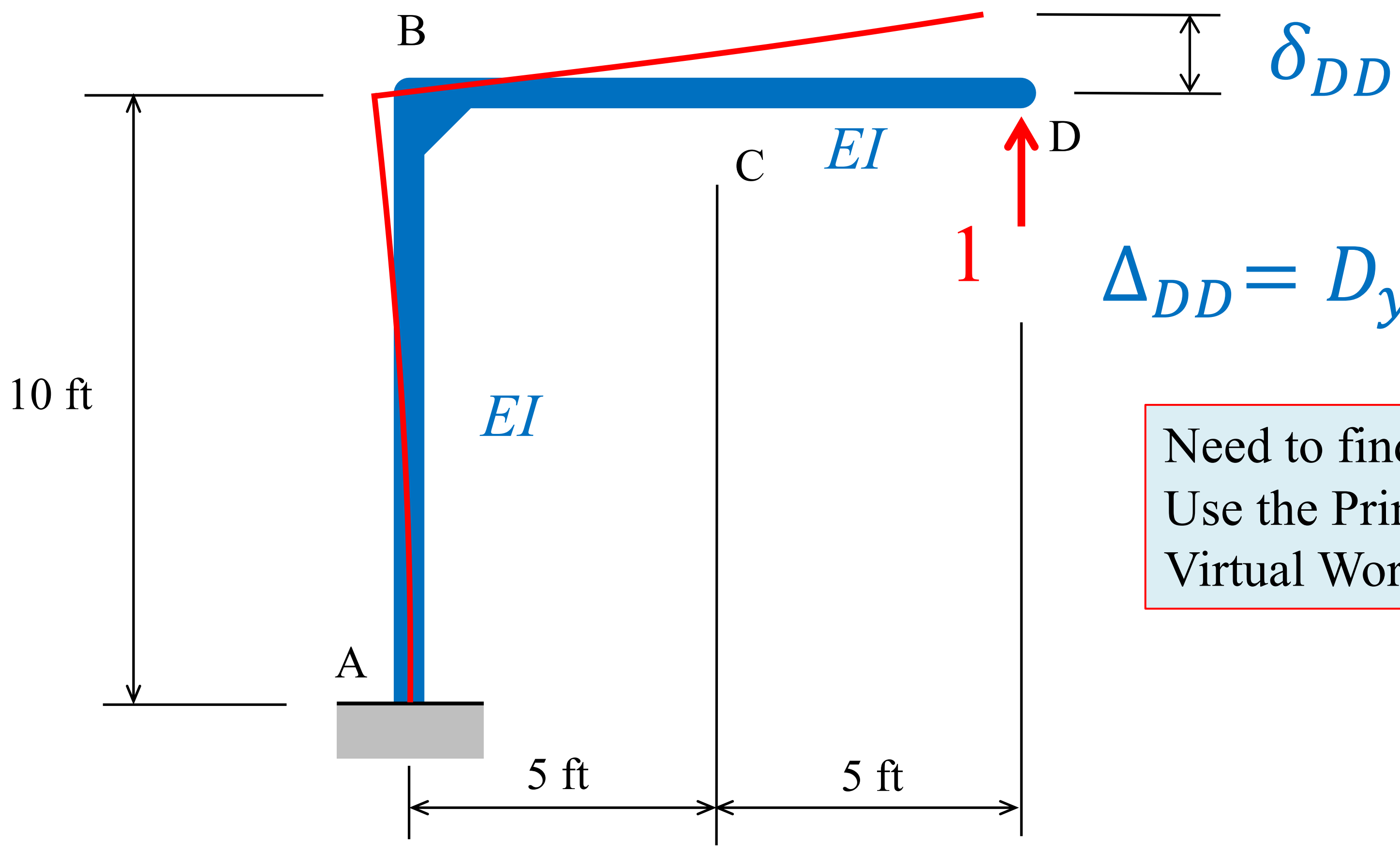


$$\Delta_{DD} = D_y \delta_{DD}$$

redundant for each redundant problem.

- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

Flexibility Coefficient

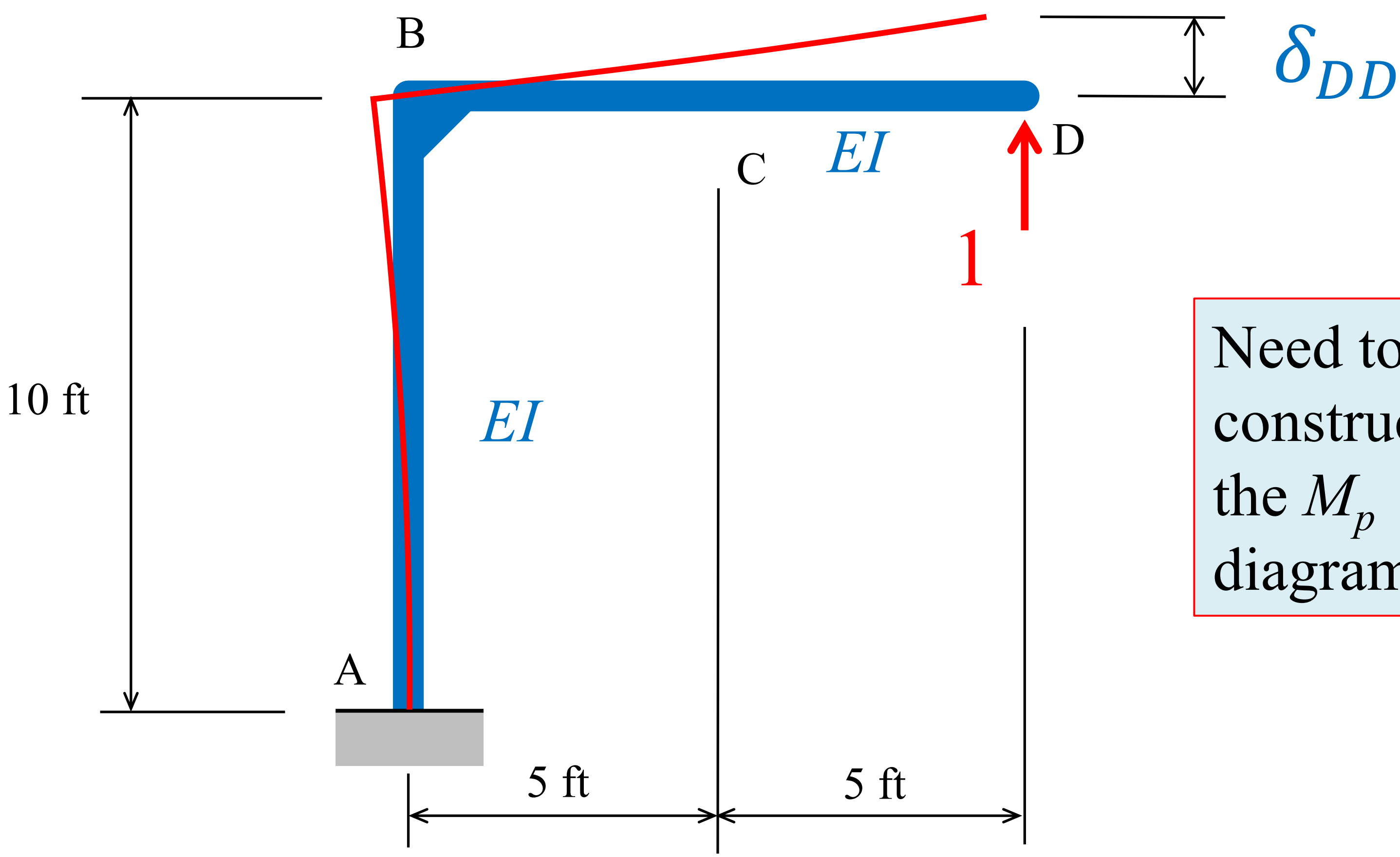


$\Delta_{DD} = D_y \delta_{DD}$

Need to find δ_{DD}
Use the Principle of Virtual Work

Solve the Flexibility Coefficient Problem

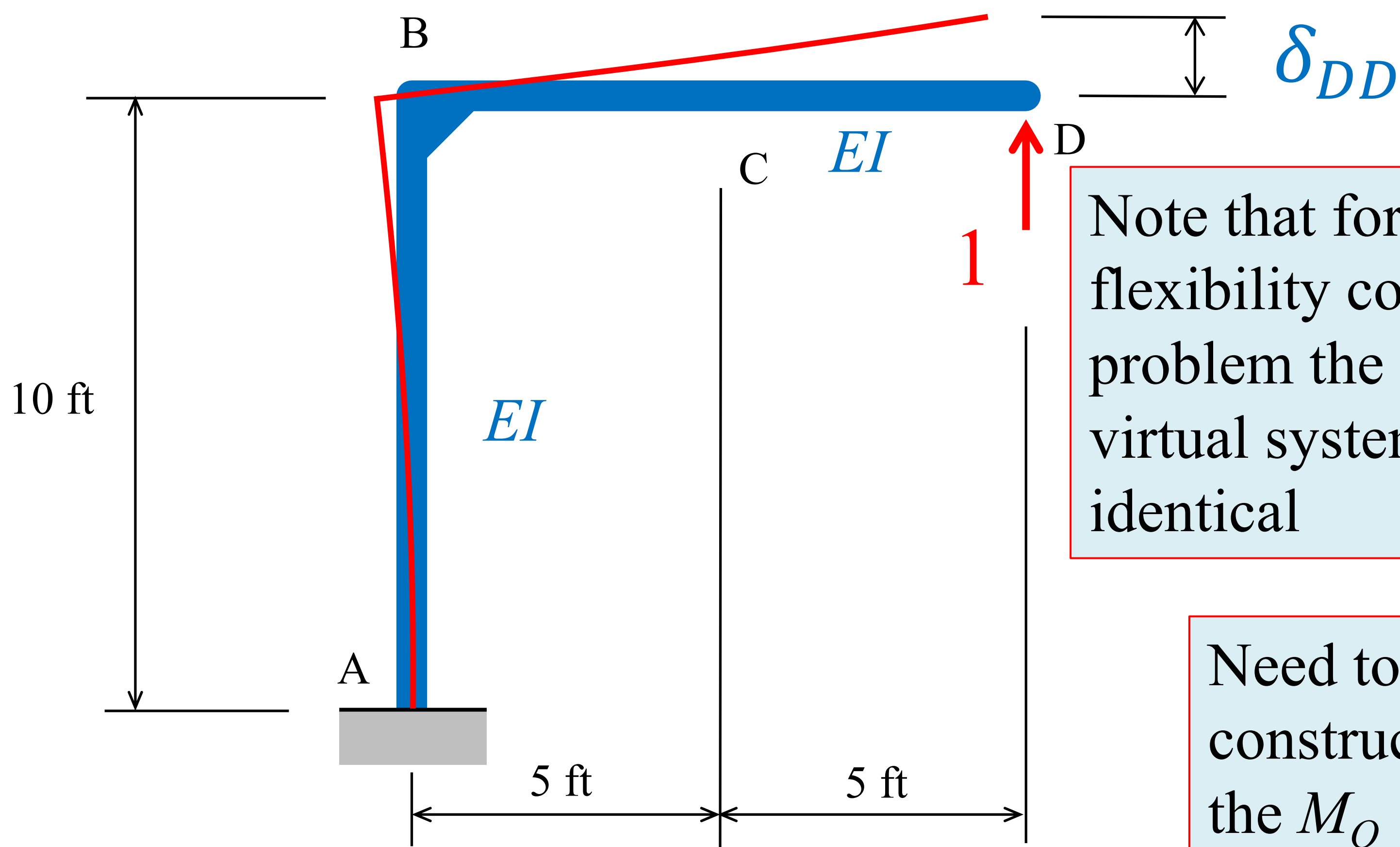
Real System



Need to construct the M_p diagram

Solve the Flexibility Coefficient Problem

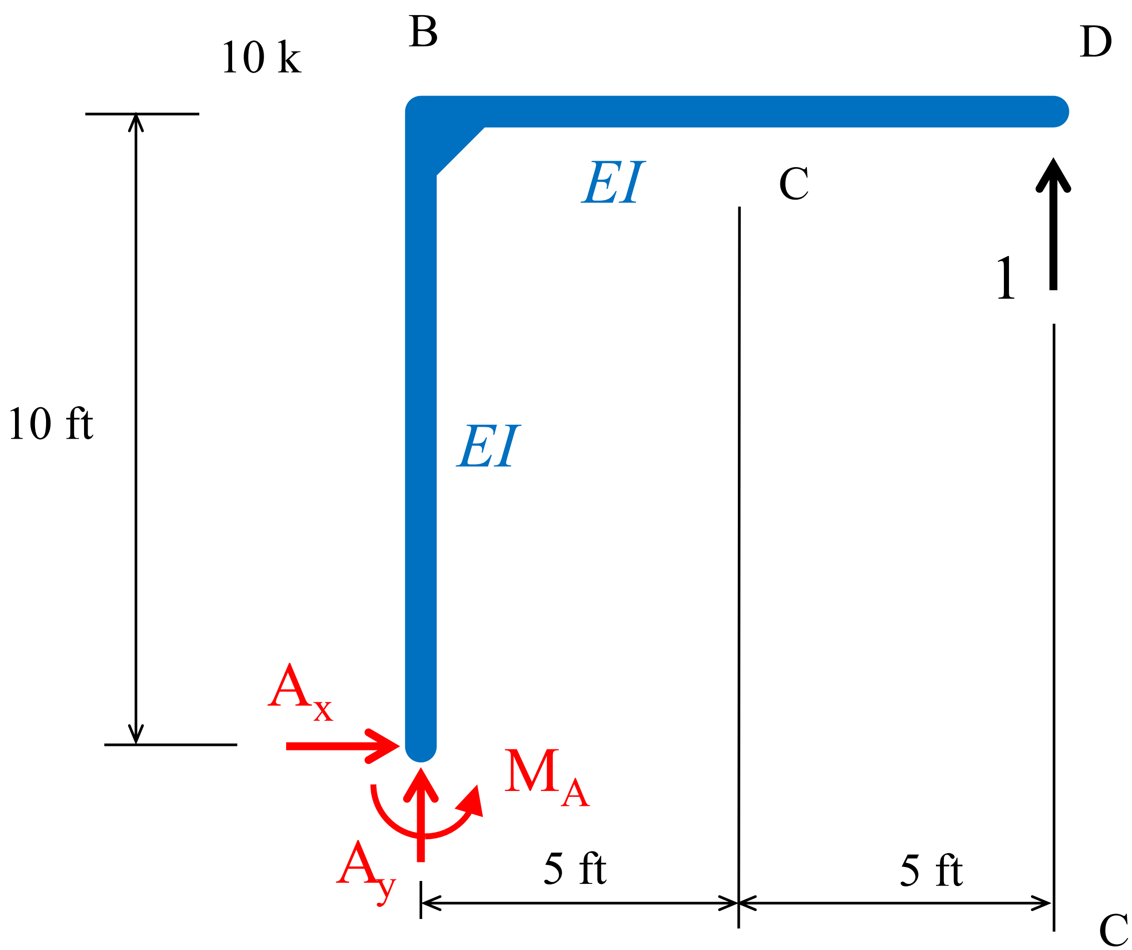
Virtual System to measure δ_{DD}



Note that for the flexibility coefficient problem the real and virtual systems are identical

Need to construct the M_Q diagram

FBD of the Flexibility Coefficient Problem



$$\circlearrowleft + \sum M_A = 0$$

$$M_A = 10 \text{ ft}$$

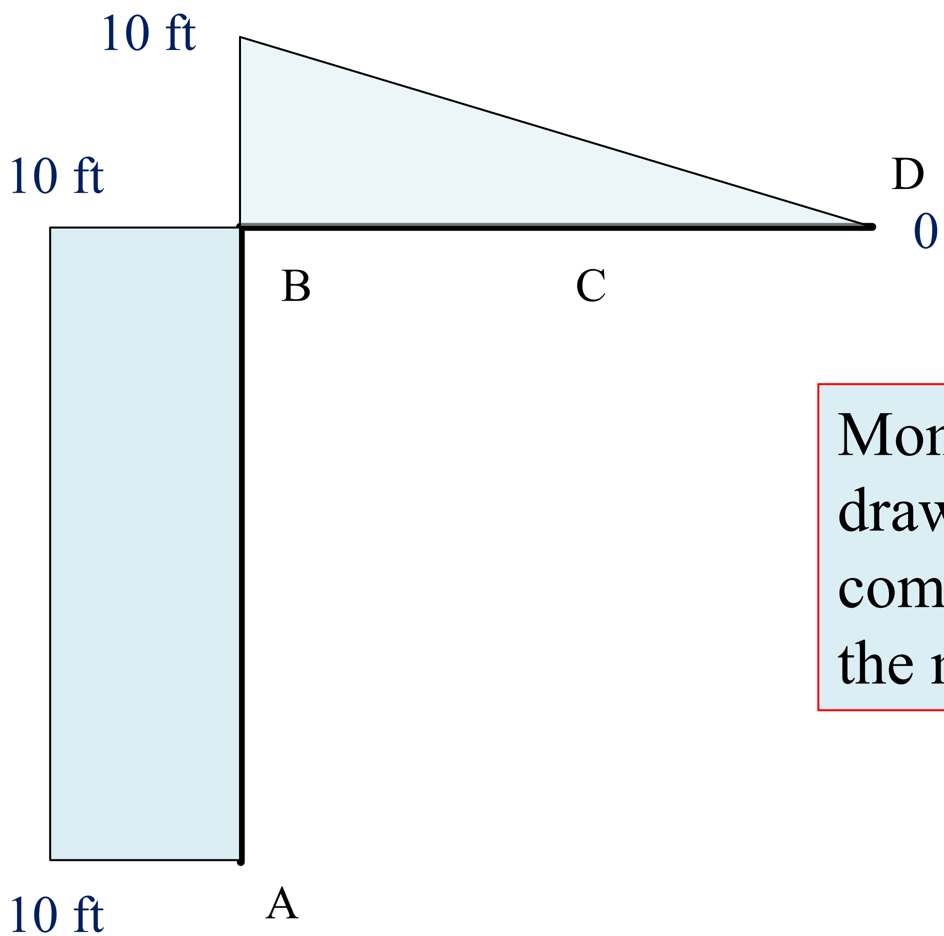
$$\xrightarrow{+} \sum F_x = 0$$

$$A_x = 0$$

$$+\uparrow \sum F_y = 0$$

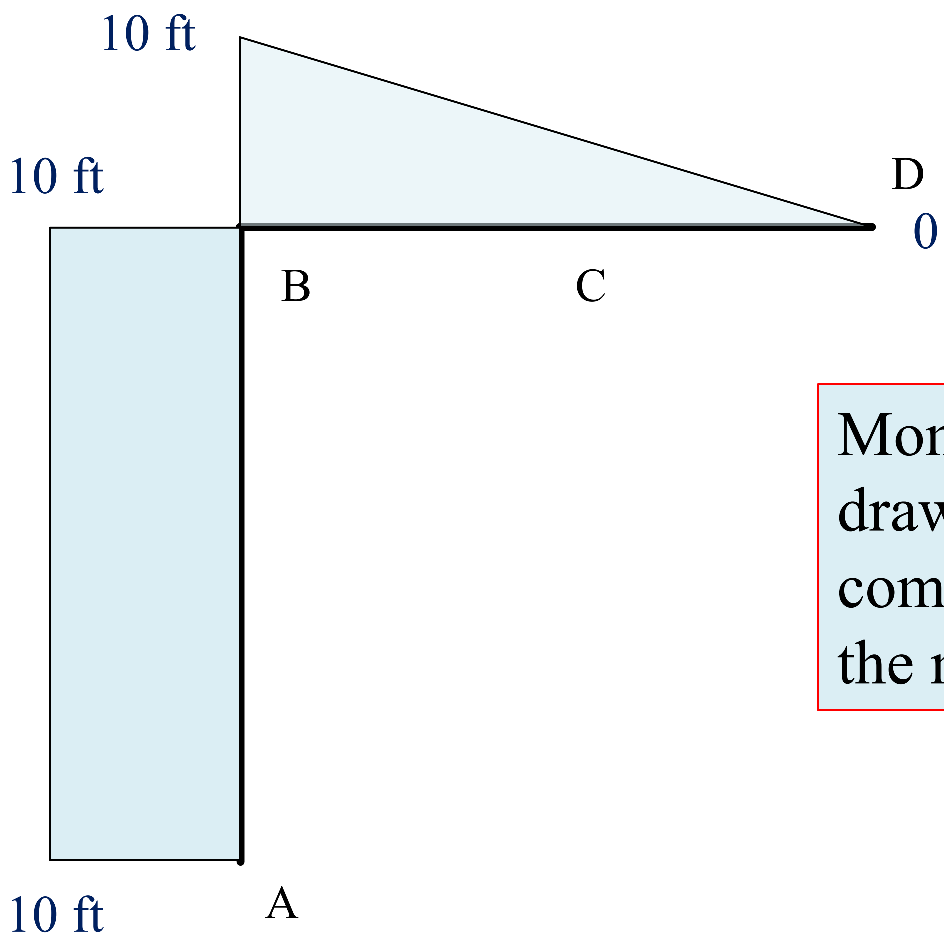
$$A_y = -1$$

M_P Diagram for the Flexibility Coefficient Problem



Moment diagram is drawn on the compression side of the member

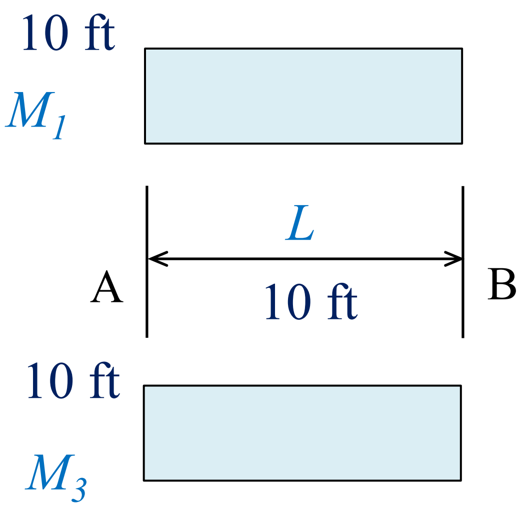
M_Q Diagram for the Flexibility Coefficient Problem

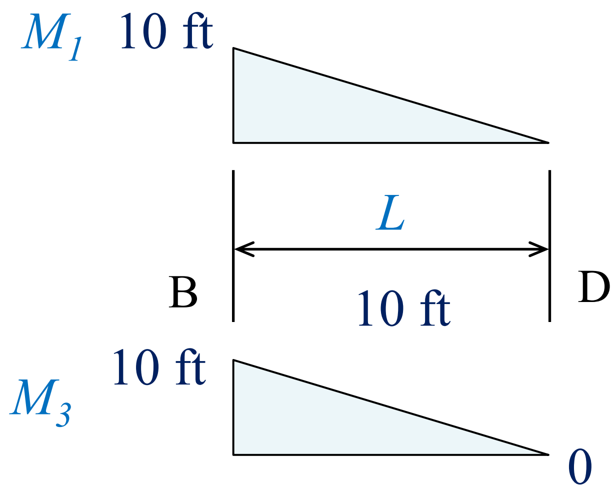


Moment diagram is drawn on the compression side of the member

Solve the Flexibility Coefficient Problem

$$1 \cdot \delta_{DD} = \frac{1}{EI}\int_0^L M_Q M_P \, dx$$





$$\delta_{DD} = \frac{1333\ \text{ft}^3}{EI}$$

$$\Delta_{DD} = D_y \delta_{DD}$$

$$\Delta_{DD} = D_y \left(\frac{1333\ \text{ft}^3}{EI}\right)$$

$M_1 M_3 L$

$(10\ \text{ft})(10\ \text{ft})(10\ \text{ft})$

$1000\ \text{ft}^3$

$\frac{1}{3} M_1 M_3 L$

$\left(\frac{1}{3}\right)(10\ \text{ft})(10\ \text{ft})(10\ \text{ft})$

$333.333\ \text{ft}^3$

Compatibility Equation at Point D

Compatibility at Point D

$$\Delta_D + \Delta_{DD} = 0$$

Compatibility Equation in terms of Redundant and Flexibility Coefficient

$$\Delta_D + D_y \delta_{DD} = 0$$

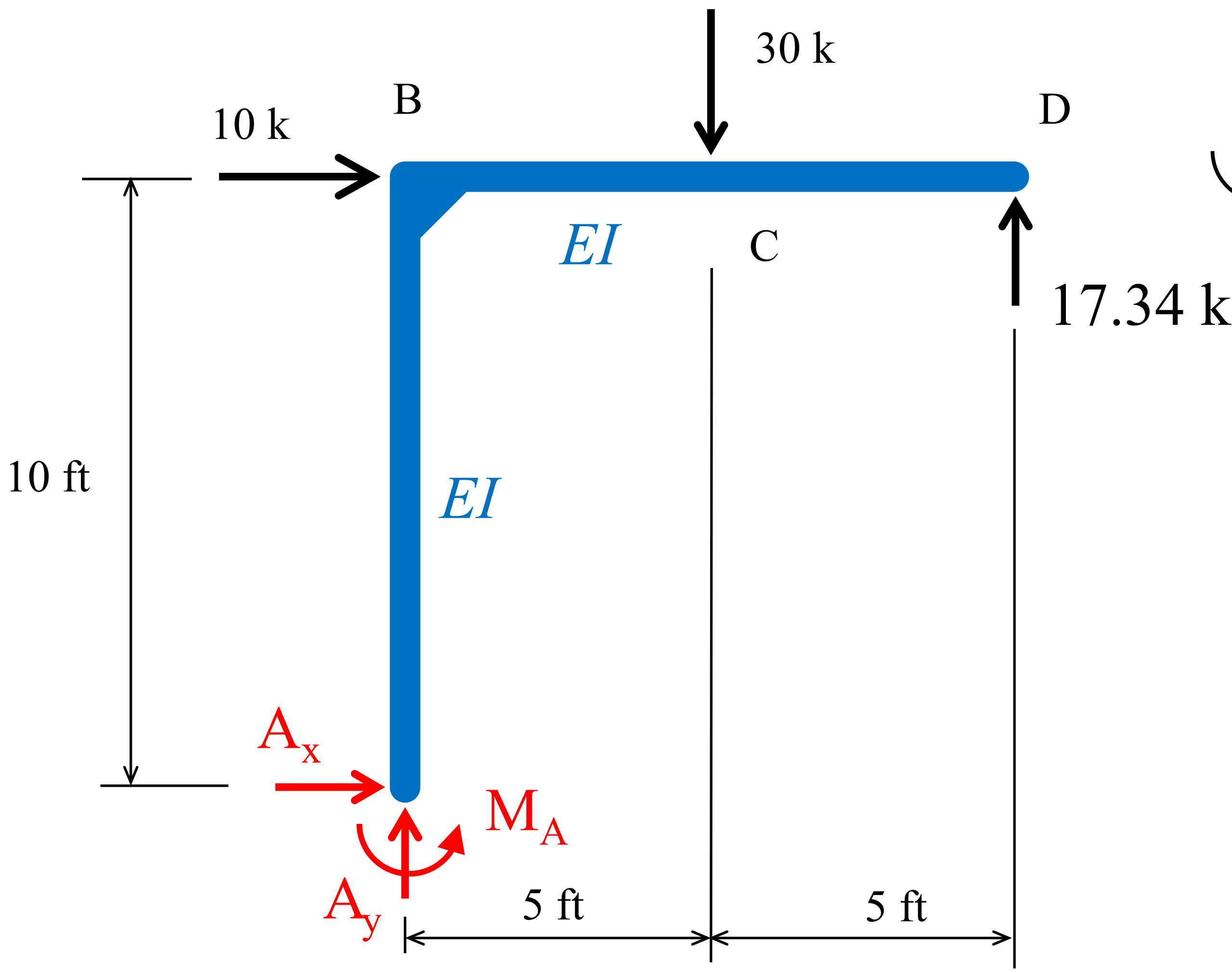
$$-\frac{23{,}125 \text{ k–ft}^3}{EI} + D_y\left(\frac{1333 \text{ ft}^3}{EI}\right) = 0$$

Solve for D_y

$$D_y = \frac{23{,}125 \text{ k–ft}^3}{EI}\left(\frac{EI}{1333 \text{ ft}^3}\right)$$

$$D_y = 17.34 \text{ k}$$

FBD of the Frame



$D_y = 17.34\text{ k}$

$\circlearrowleft + \sum M_A = 0$

$M_A = 76.6\text{ k-ft}$

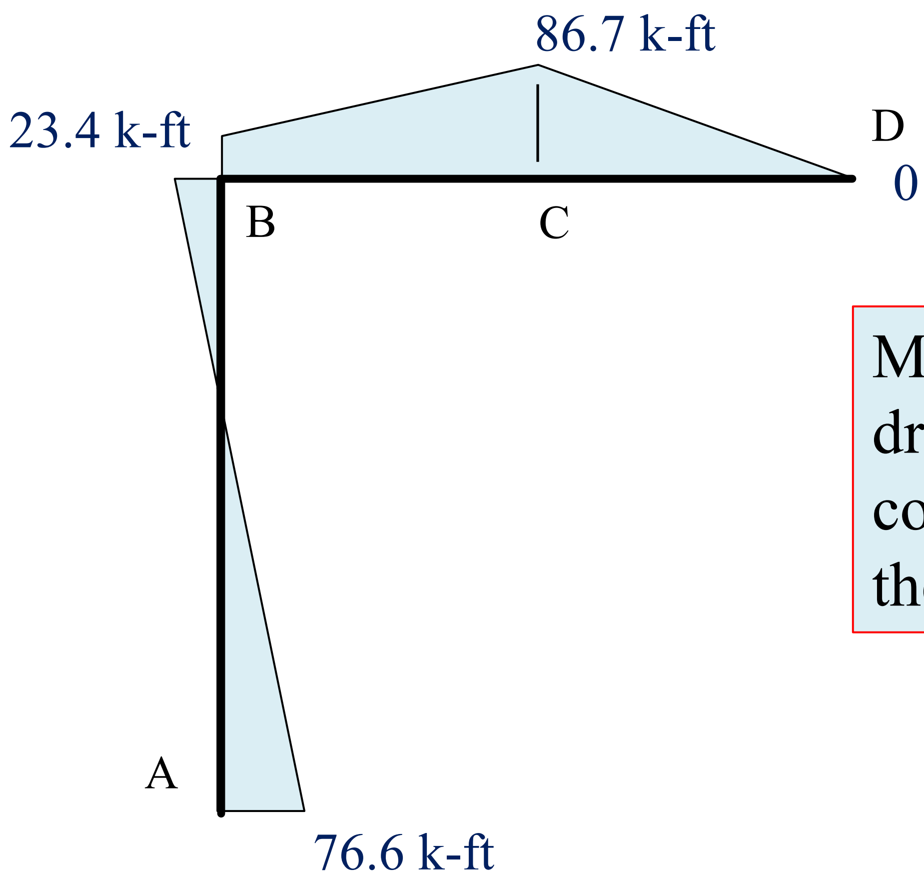
$\xrightarrow{+} \sum F_x = 0$

$A_x = -10\text{ k}$

$+\uparrow \sum F_y = 0$

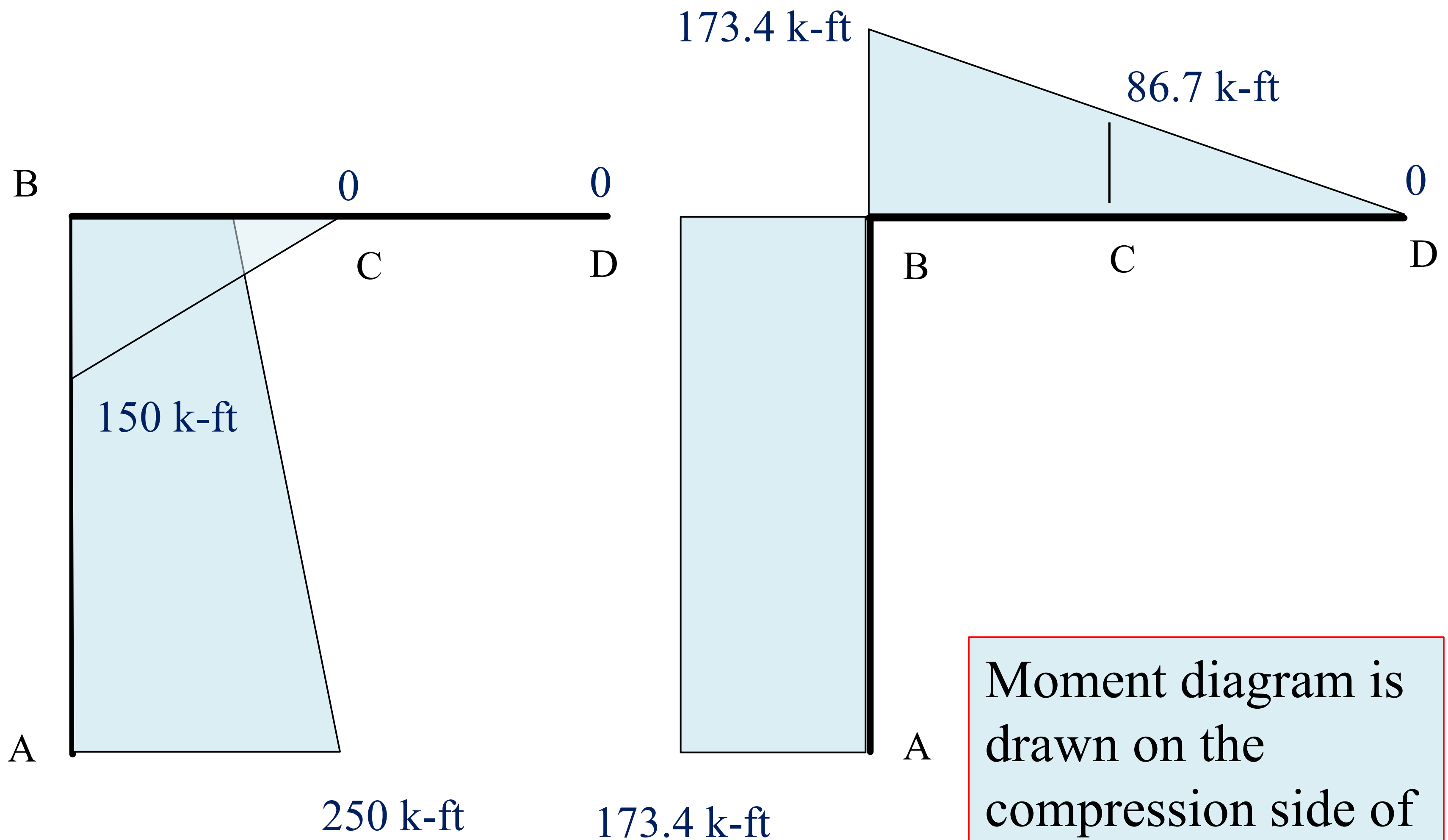
$A_y = 12.66\text{ k}$

Moment Diagram for the Frame



Moment diagram is drawn on the compression side of the member

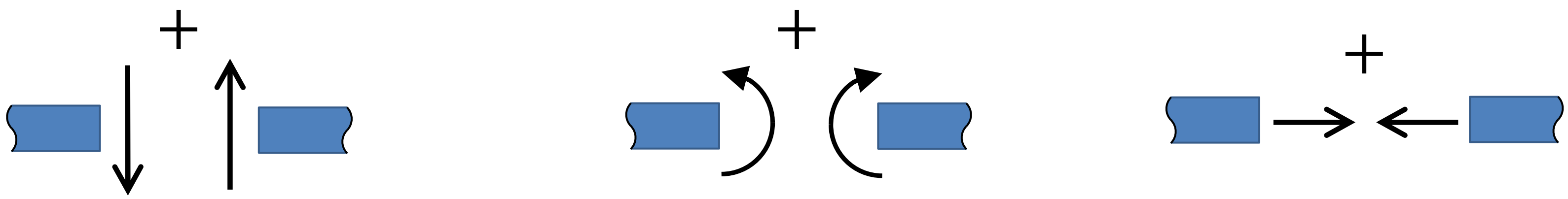
Moment Diagrams for the Primary and Redundant Problems



Moment diagram is drawn on the compression side of the member

Choose sign convention for internal forces for both horizontal and vertical members

For horizontal member BDE



For vertical member ABC