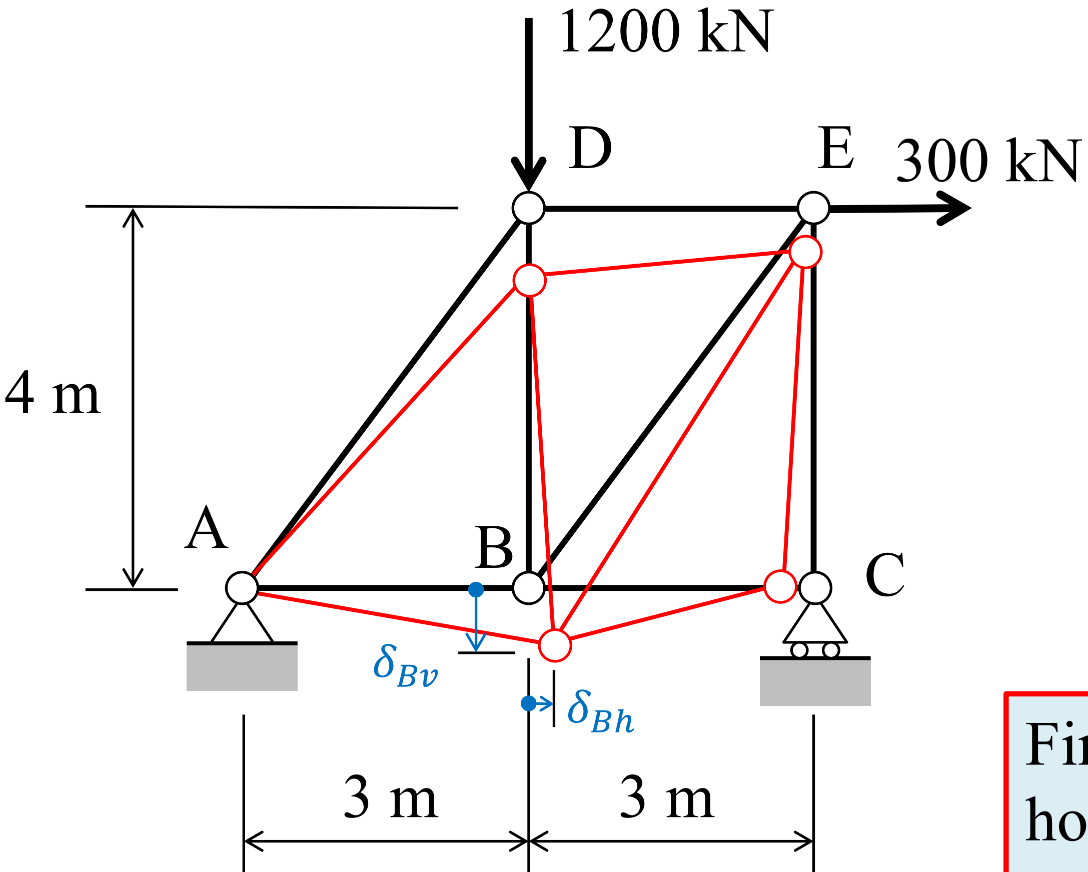
Virtual Work Truss Example
Loads to Truss Joints

Steven Vukazich
San Jose State University

Example Using the Principle of Virtual Work

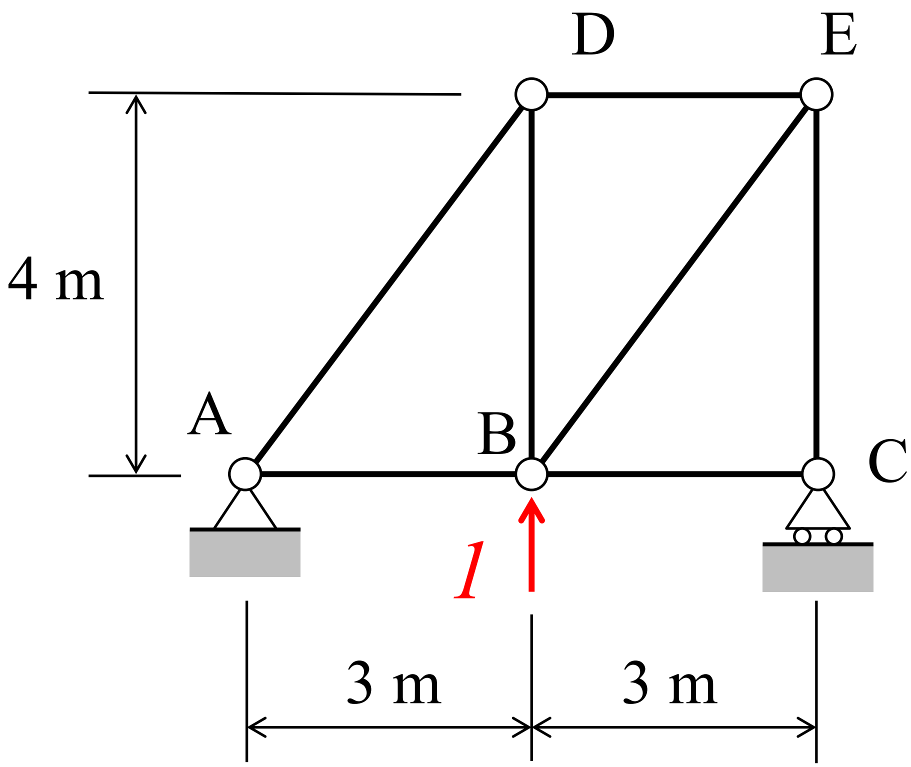


Consider the idealized truss structure with a pin support at A and a roller support at C. The truss is subjected to applied loads at D and E.

For all truss members use:
$A = 25\ \text{cm}^2$
$E = 210\ \text{GPa}$

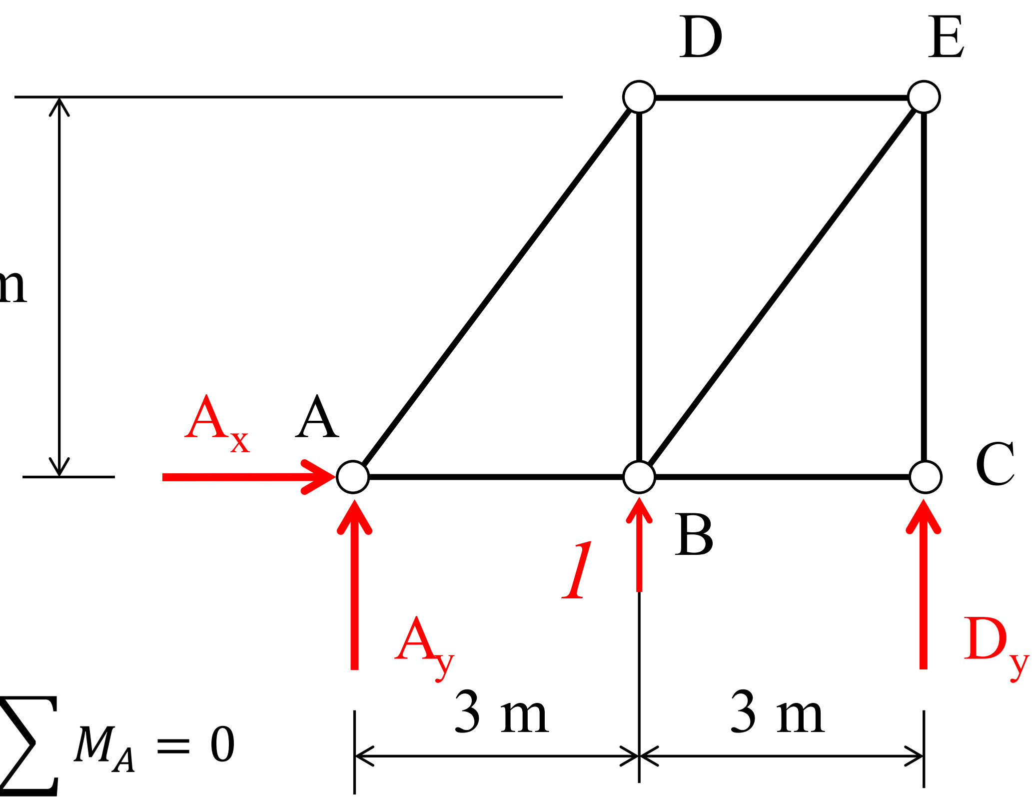
Find the vertical and the horizontal displacement of point B using the Principle of Virtual Work

Virtual System to Measure δ_{Bv}



1. Remove all loads from the structure;
2. Apply a unit, dimensionless virtual load **in-line** with the real displacement, δ_{Bv}, that we want to find;
3. Perform a truss analysis to find all truss member virtual axial forces, F_{Qi}

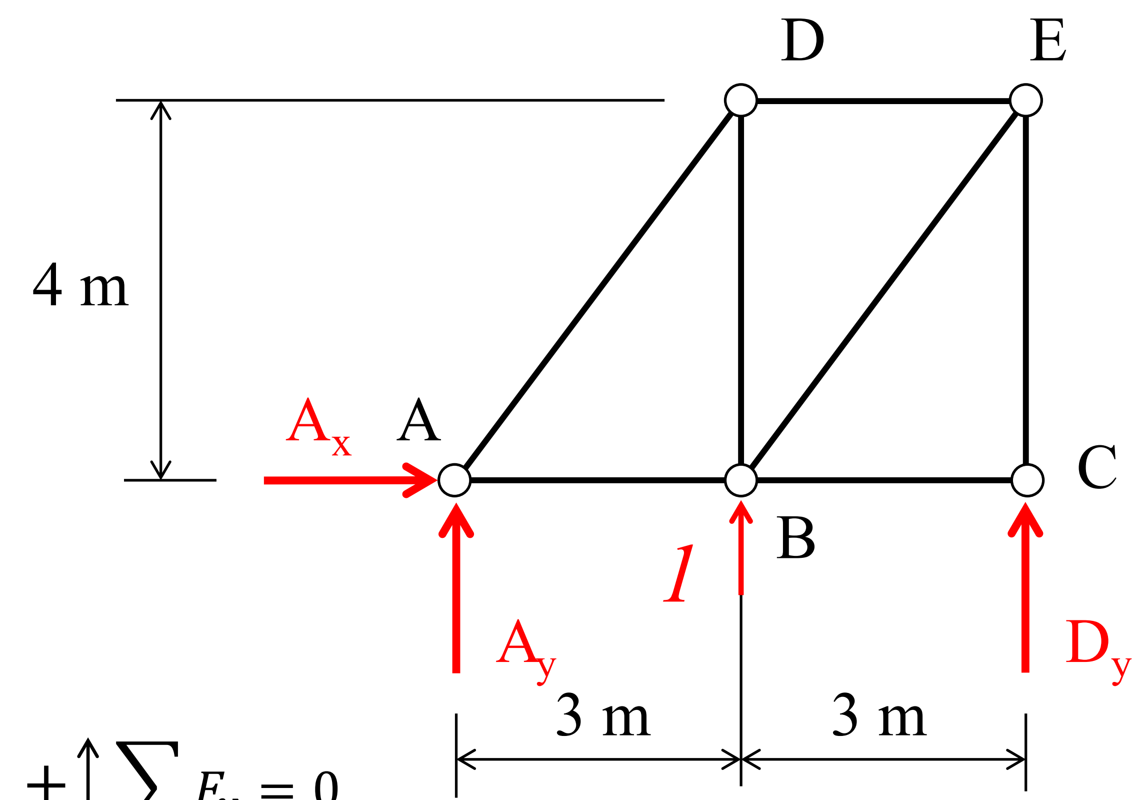
Find Support Reactions



$$\circlearrowleft + \sum M_A = 0$$

$D_y = -0.5$

Find Support Reactions

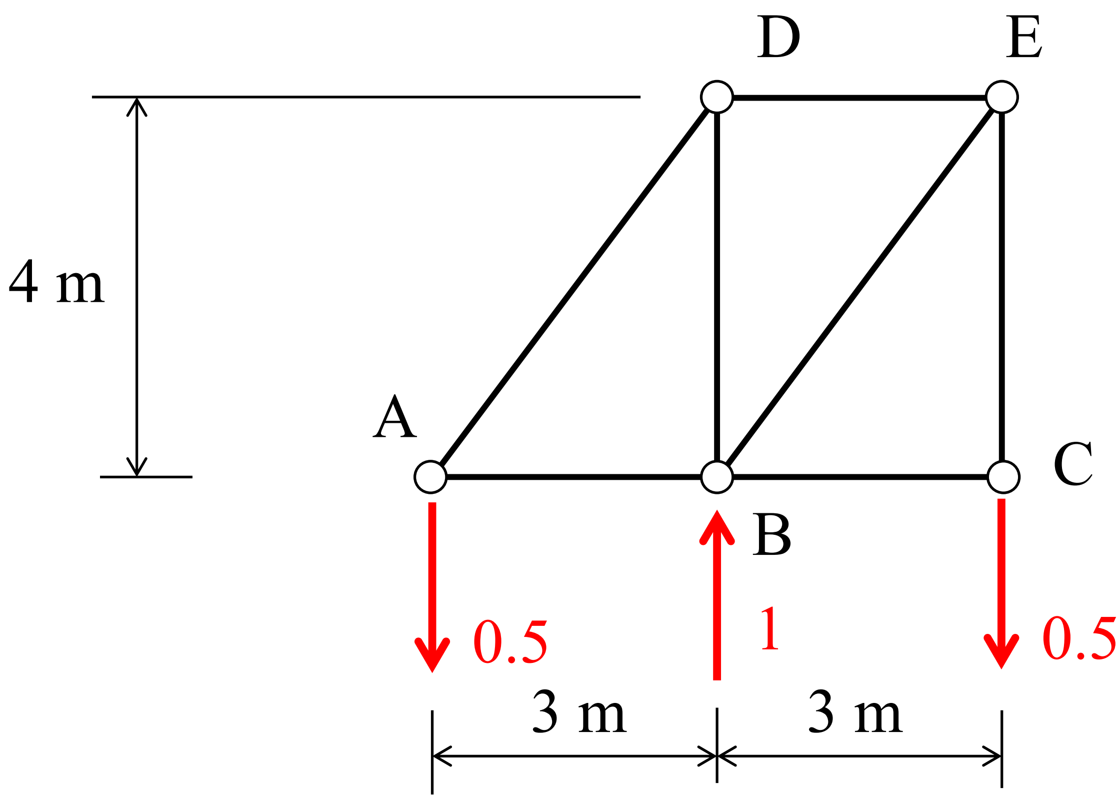


$$+\uparrow \sum F_y = 0$$

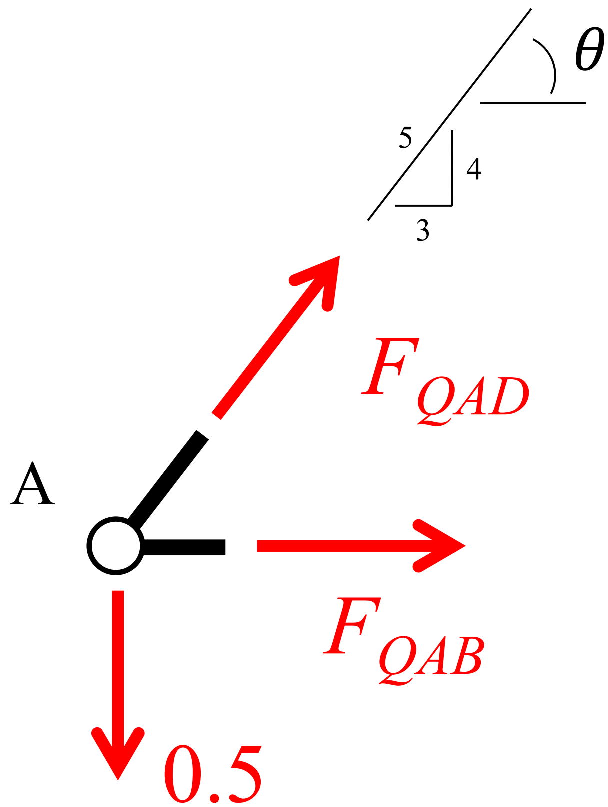
$$\xrightarrow{+} \sum F_x = 0$$

$A_y = -0.5$

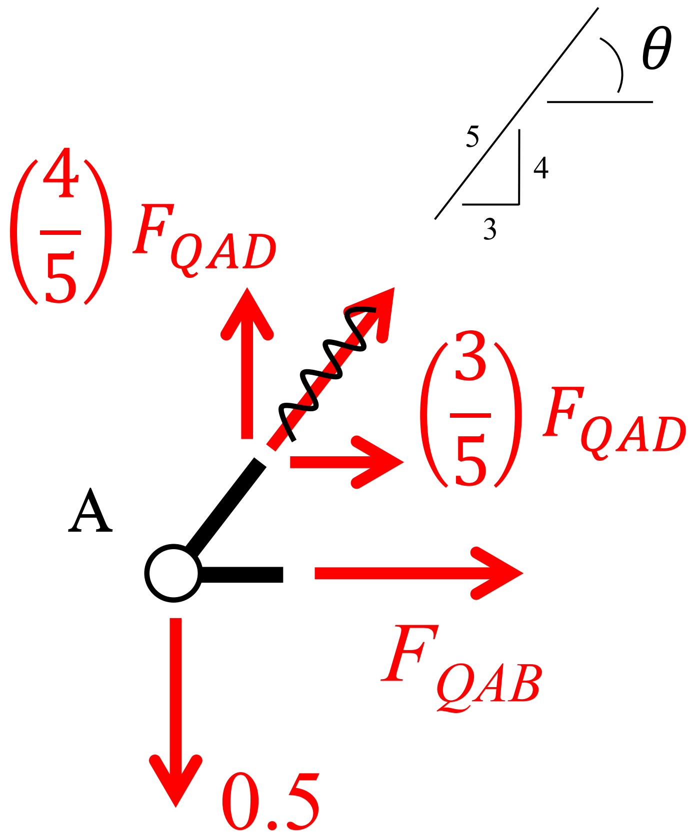
Virtual System Support Reactions



FBD of Joint A



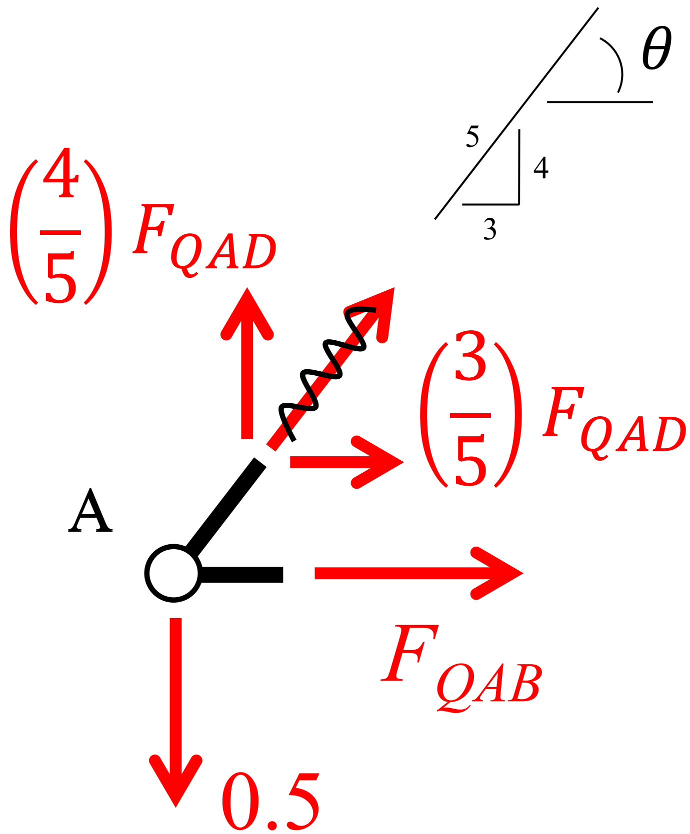
FBD of Joint A



$$+\uparrow \sum F_y = 0$$

$F_{QAD} = 0.625$

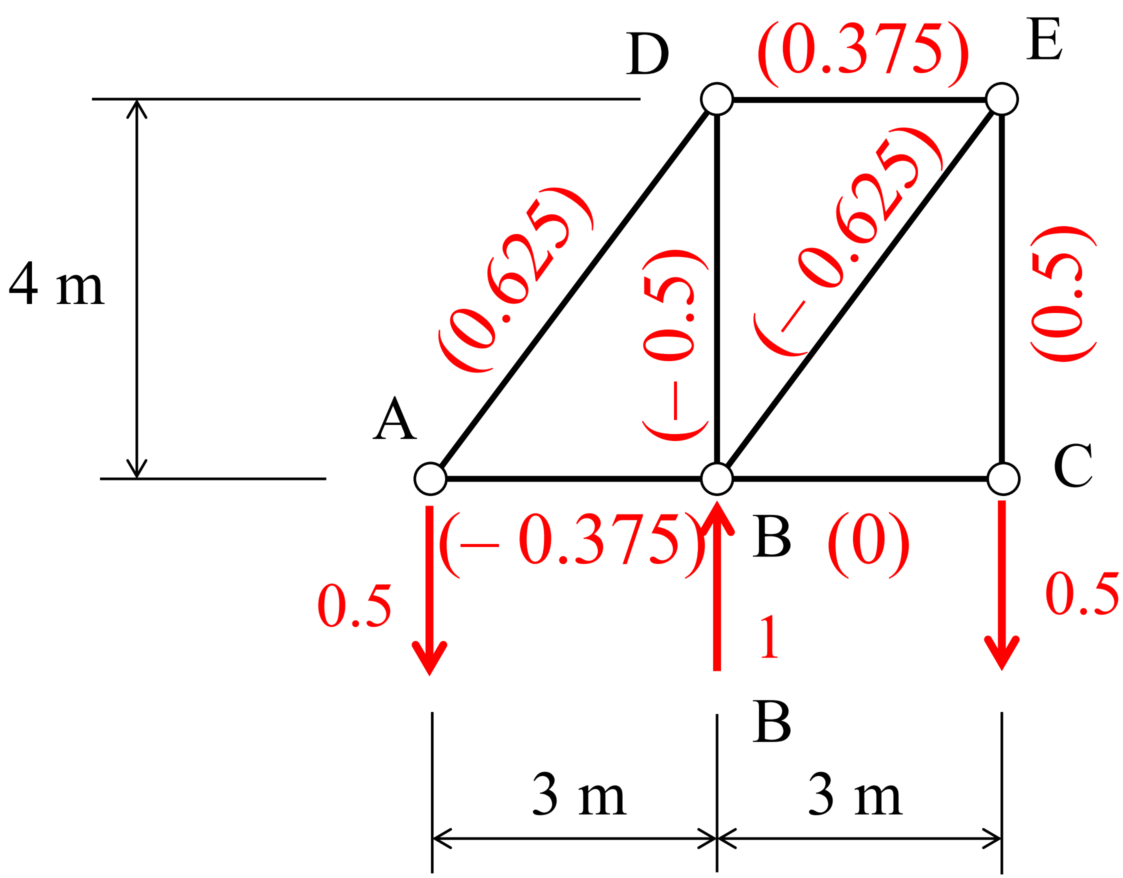
FBD of Joint A



$$\xrightarrow{+} \sum F_x = 0$$

$F_{QAB} = -0.375$

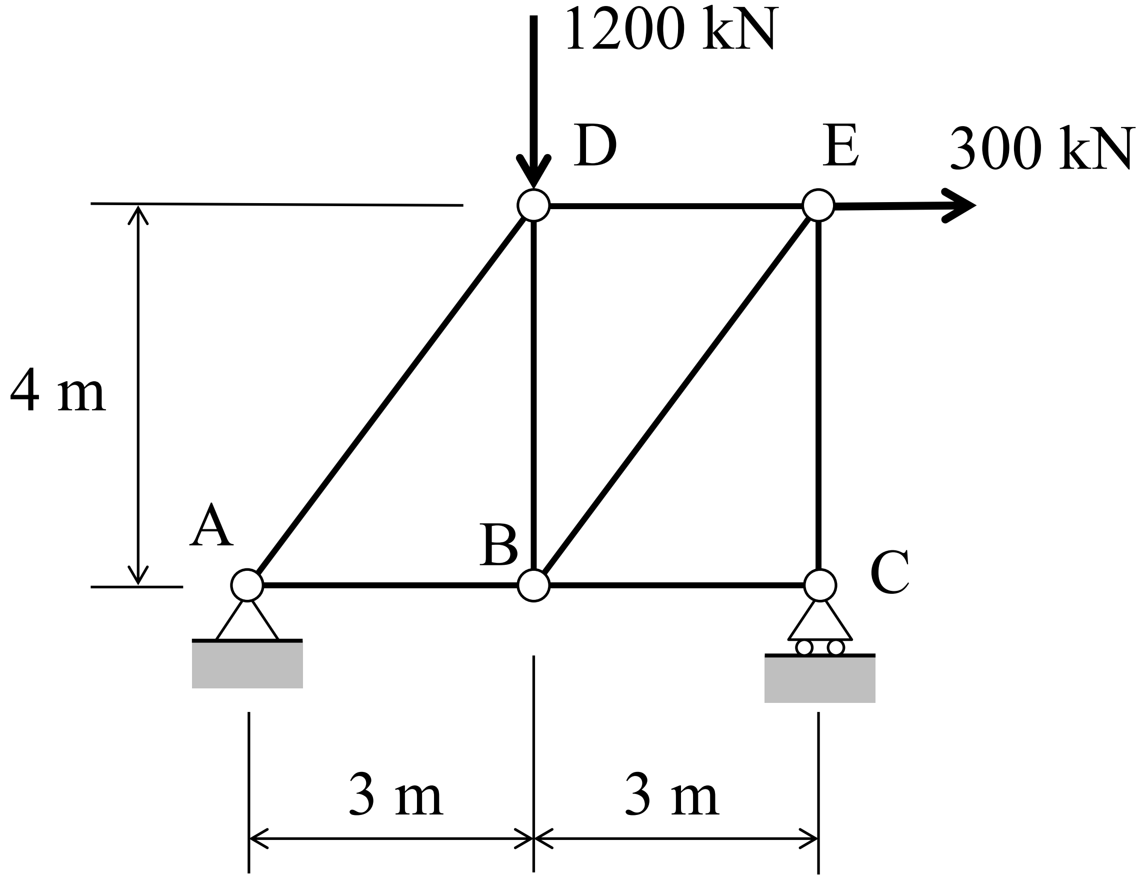
Virtual System Results on a FBD of the Entire Truss



Virtual truss member forces, F_{Qi}

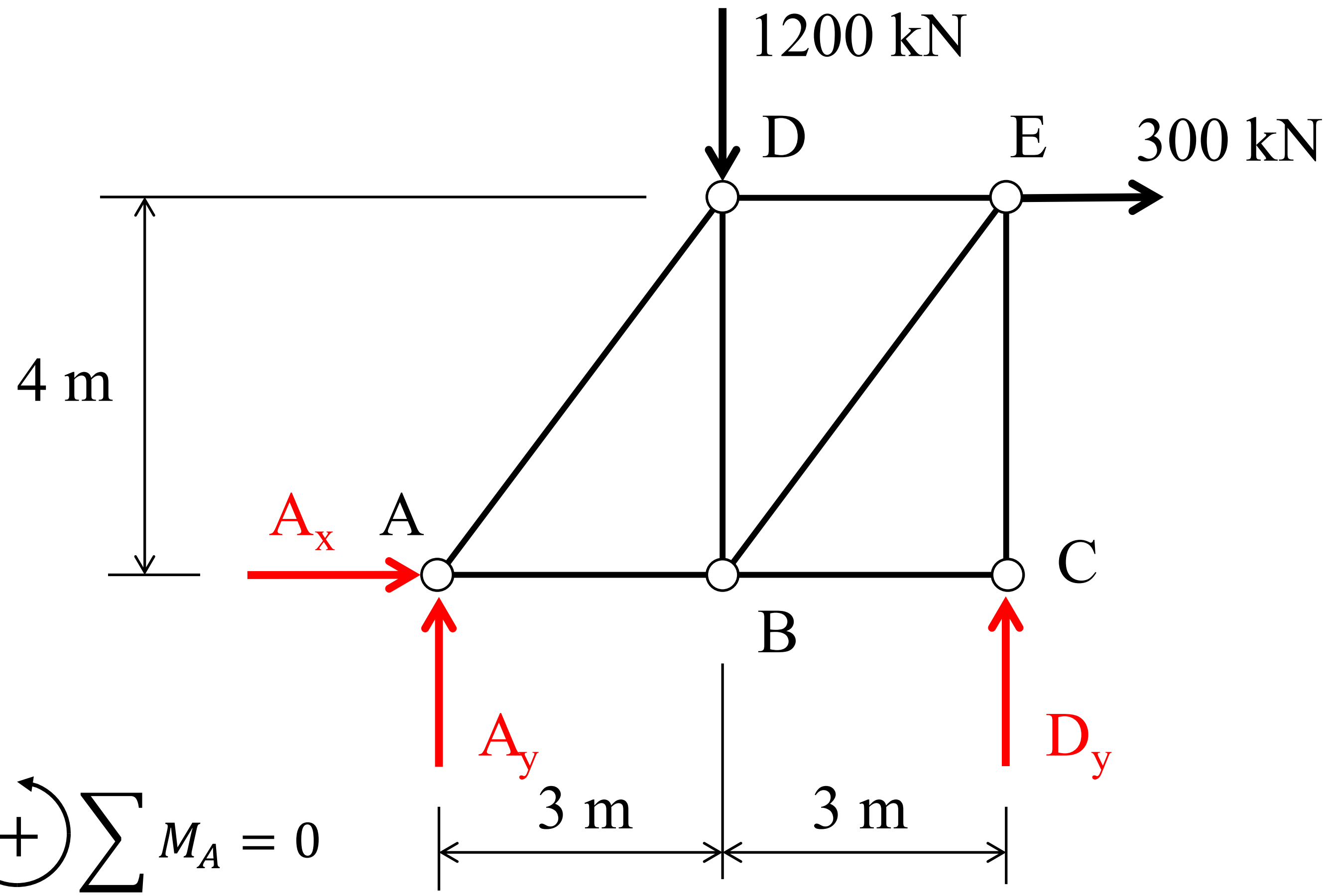
Tension is Positive

Step 2 –Real Analysis



1. Place all of the loads on the structure;
2. Perform a truss analysis to find all truss member real axial forces, F_{Pi}

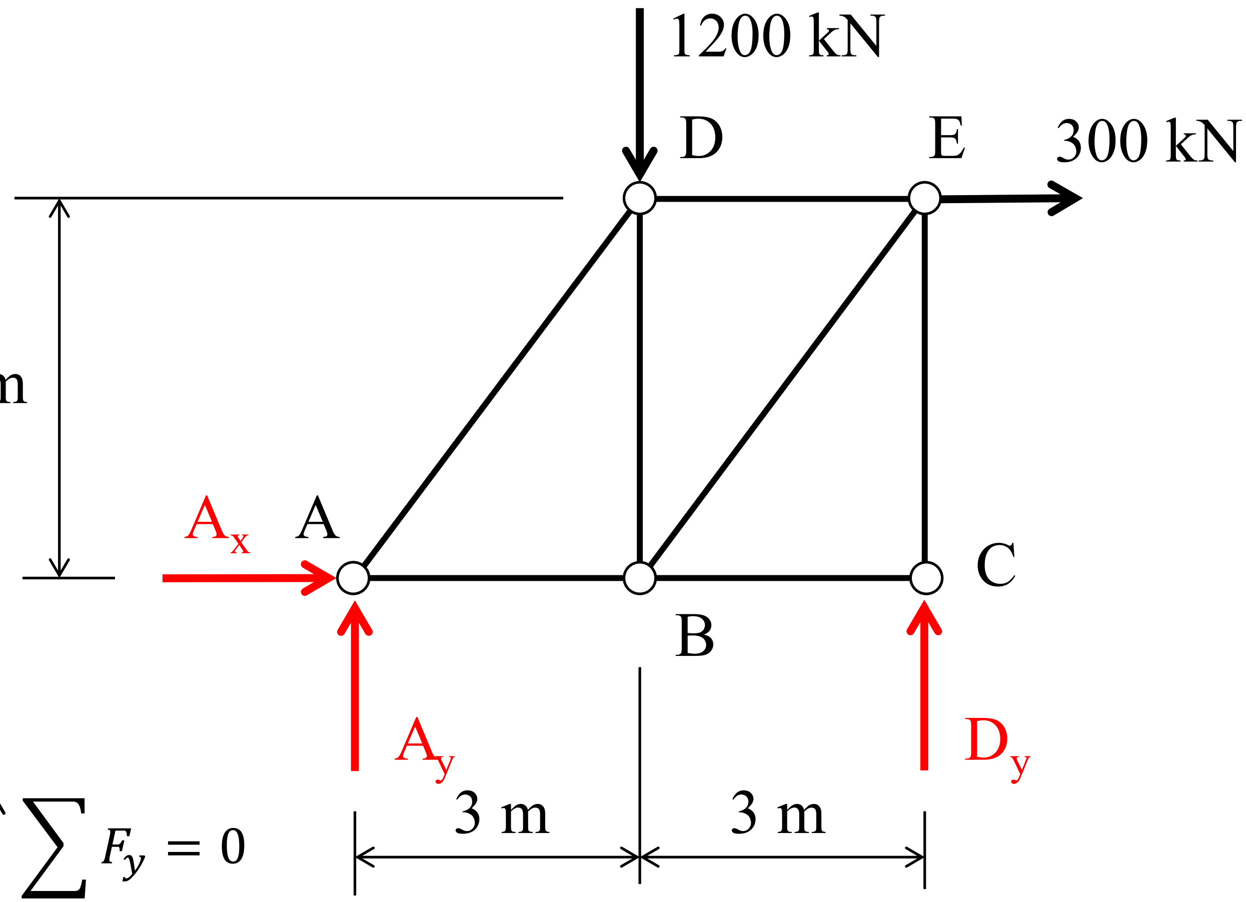
Use Equilibrium to Find Support Reactions



$$\circlearrowleft + \sum M_A = 0$$

$D_y = 800$ kN

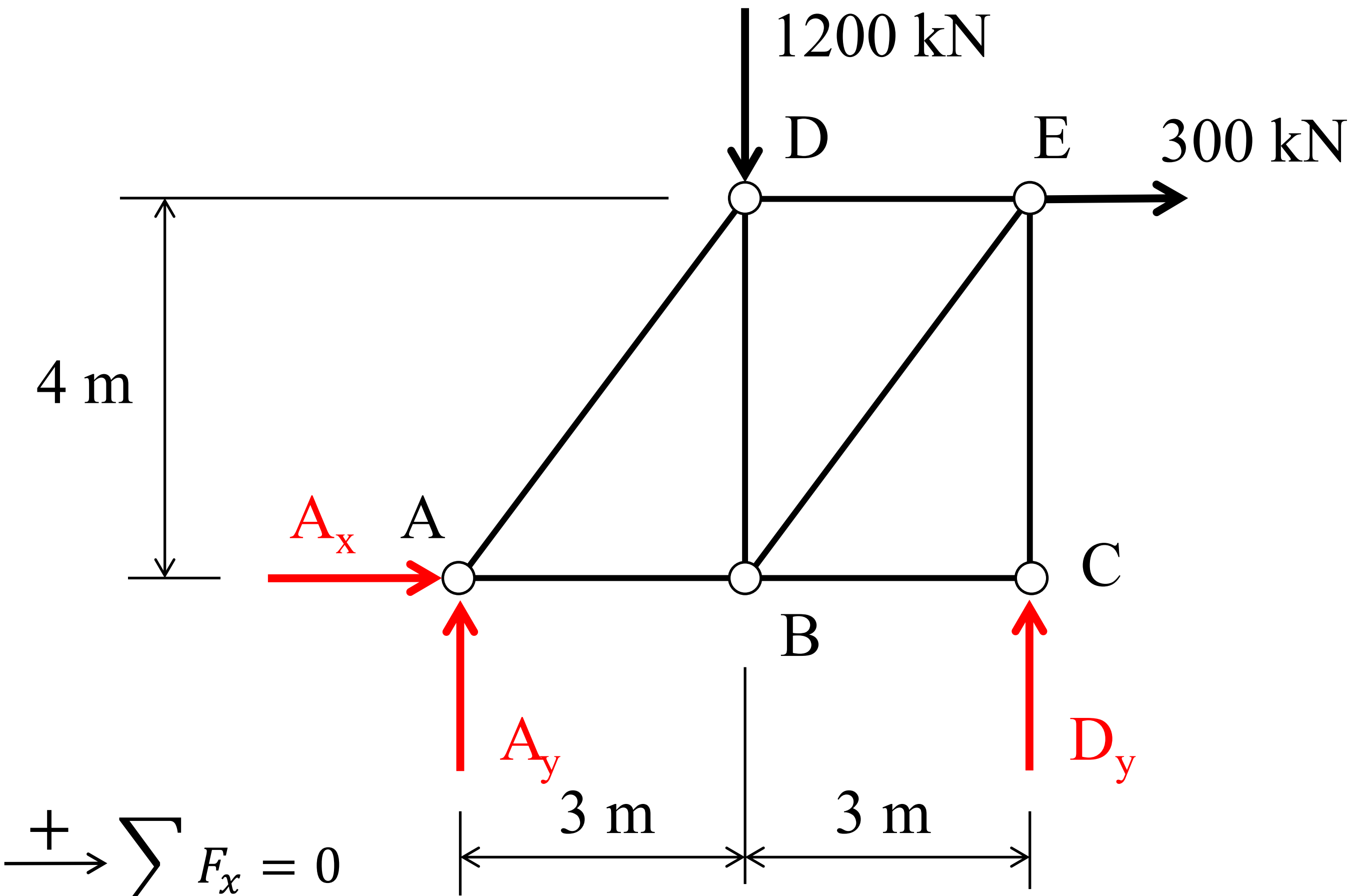
Use Equilibrium to Find Support Reactions



$$+\uparrow \sum F_y = 0$$

$A_y = 400$ kN

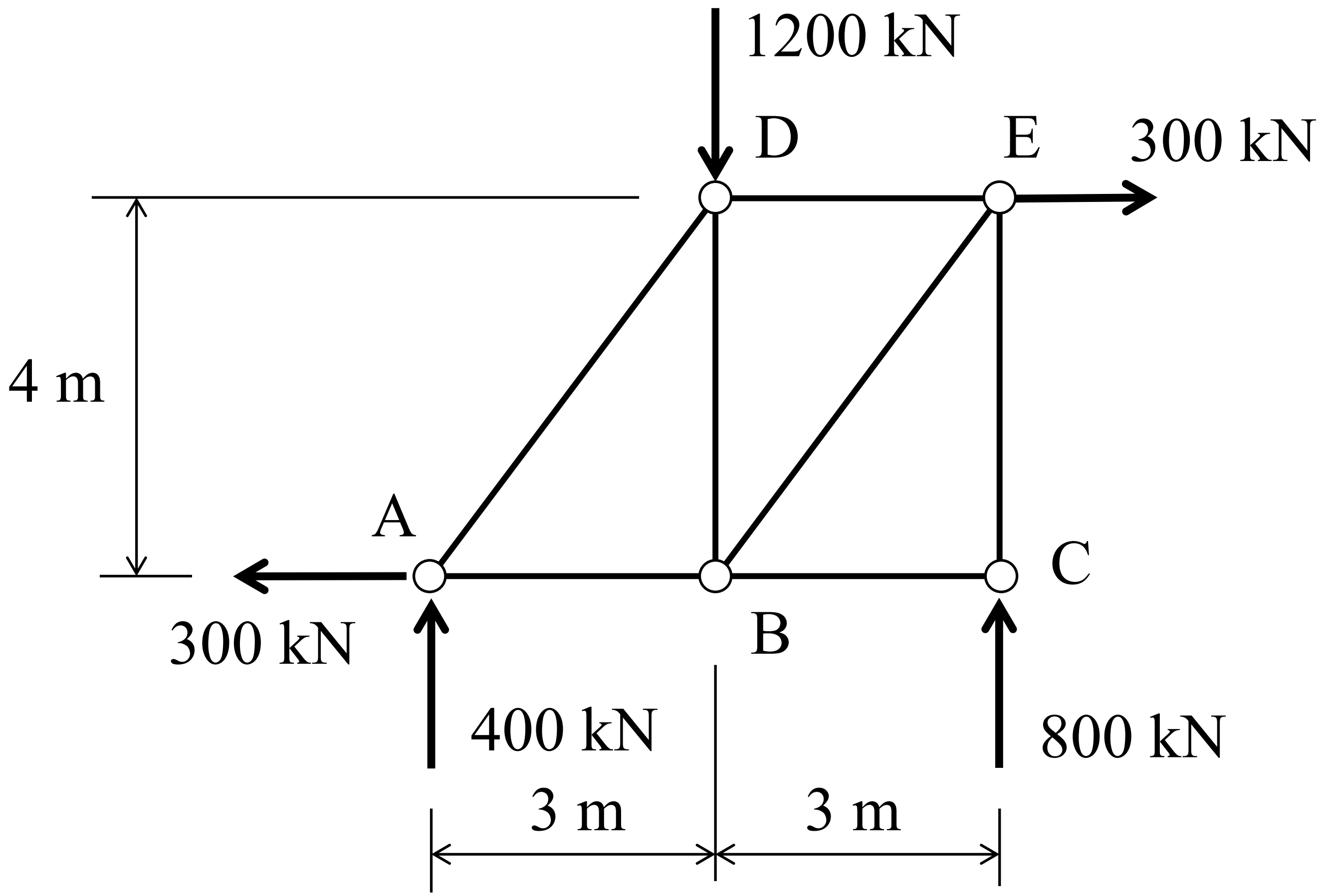
Use Equilibrium to Find Support Reactions



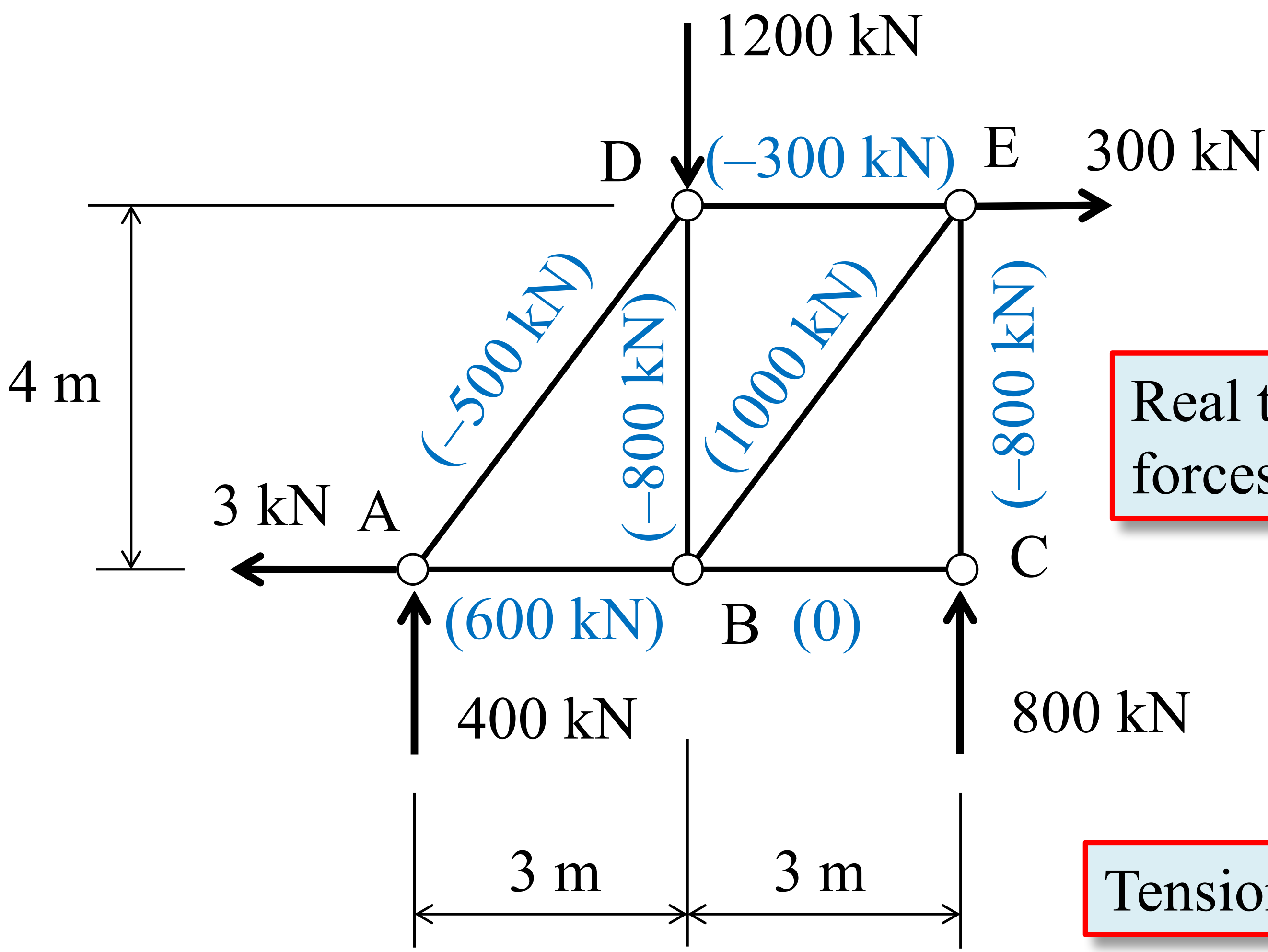
$\xrightarrow{+} \sum F_x = 0$

$A_x = -300$ kN

FBD Showing Known Support Reactions



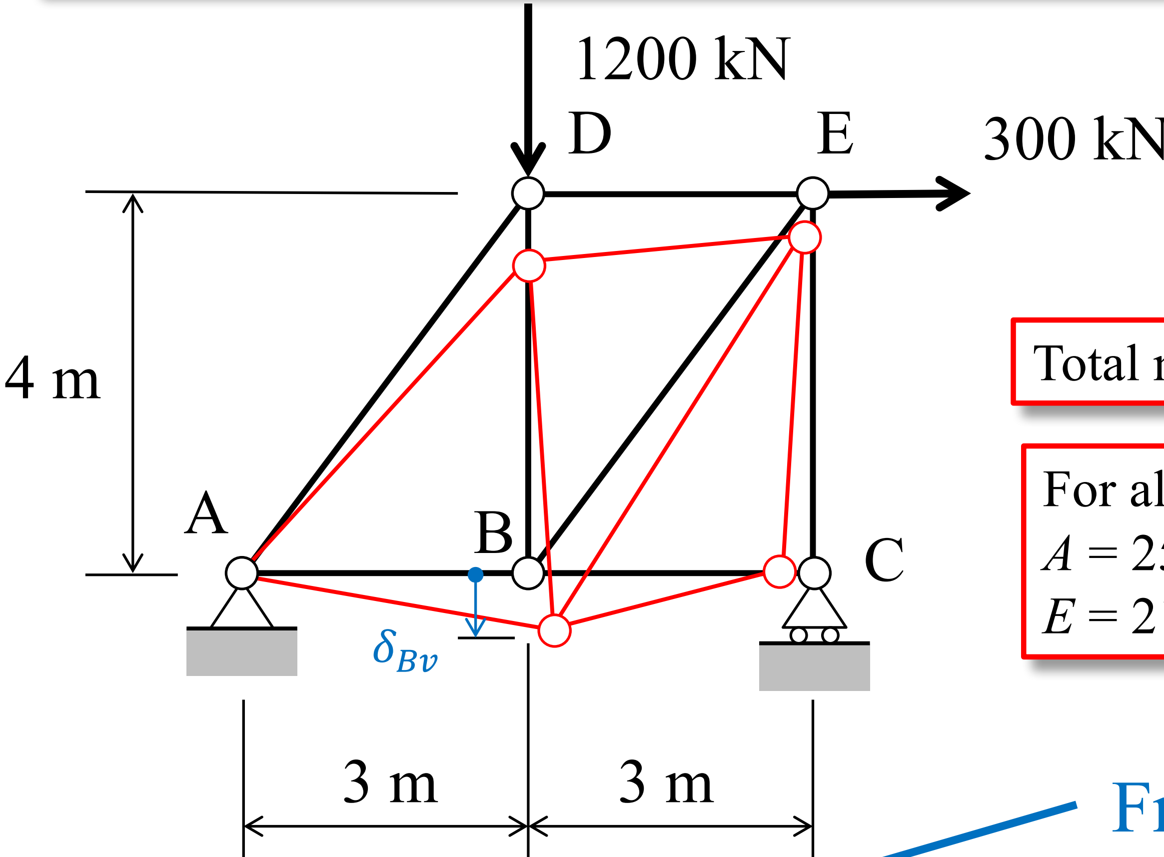
Show Results on FBD of Entire Truss



Real truss member forces, F_{Pi}

Tension is Positive

Step 3 – Use the Principle of Virtual Work to Find δ_{Bv}



Total number of truss members = 7

For all truss members use:
A = 25 cm^2
E = 210 GPa

$$1 \cdot \delta_{Bv} = \sum_{i=1}^{7} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i}$$

From Step 2 – real analysis

From Step 1 –virtual analysis

Use a Table to Organize Virtual Work Calculations

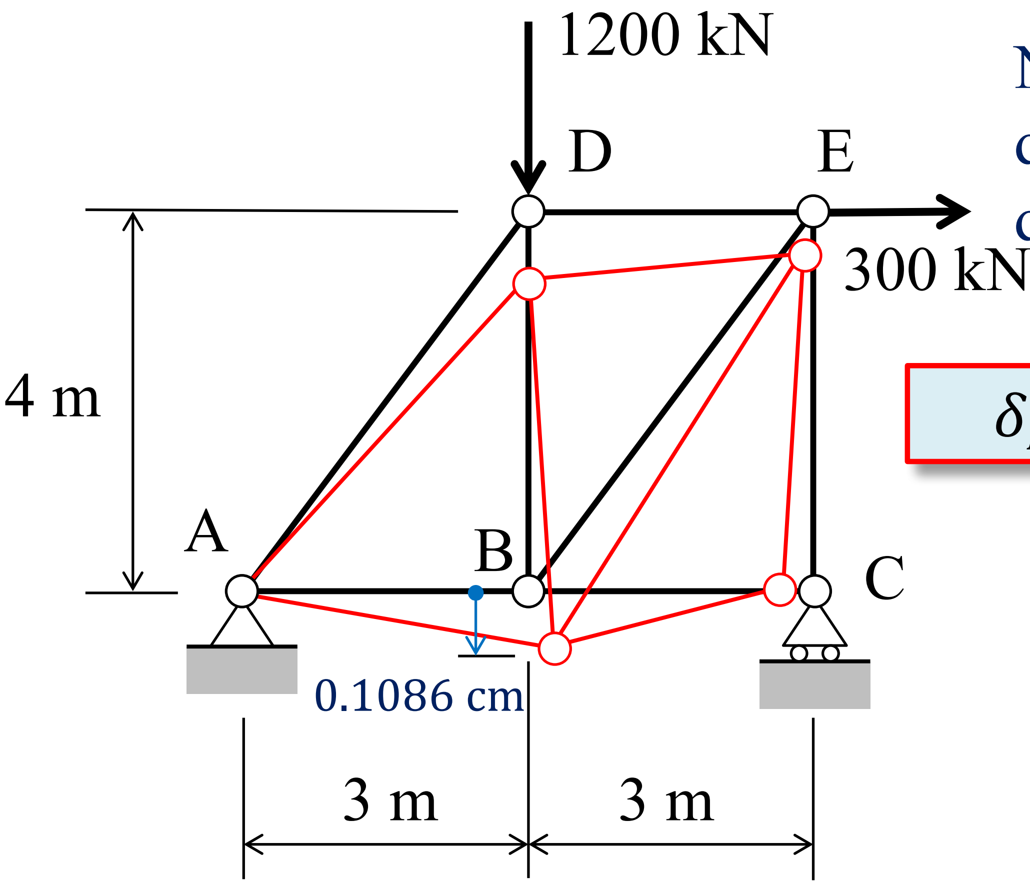
Member	A (cm^2)	E (GPa)	L (m)	F_Q	F_P(kN)	U_Q(cm)
AD	25	210	5	0.625	– 500	– 0.2976
AB	25	210	3	– 0.375	600	– 0.1286
BD	25	210	4	– 0.5	– 800	0.3048
DE	25	210	3	0.375	– 300	– 0.06429
BE	25	210	5	– 0.625	1000	– 0.5952
BC	25	210	3	0	0	0
EC	25	210	4	0.5	– 800	– 0.3048
Total						**– 1.086**

Sample Calculation

$$F_{QAD}\ \frac{F_{PAD}L_{AD}}{A_{AD}E_{AD}} = 0.625\left[\frac{(-500\ \text{kN})(5\ \text{m})\left(\frac{100\ \text{cm}}{\text{m}}\right)}{(25\ \text{cm}^2)(210\ \text{kN/mm}^2)\left(\frac{100\ \text{mm}^2}{\text{cm}^2}\right)}\right] = -0.2976\ \text{cm}$$

Results for δ_{Bv}

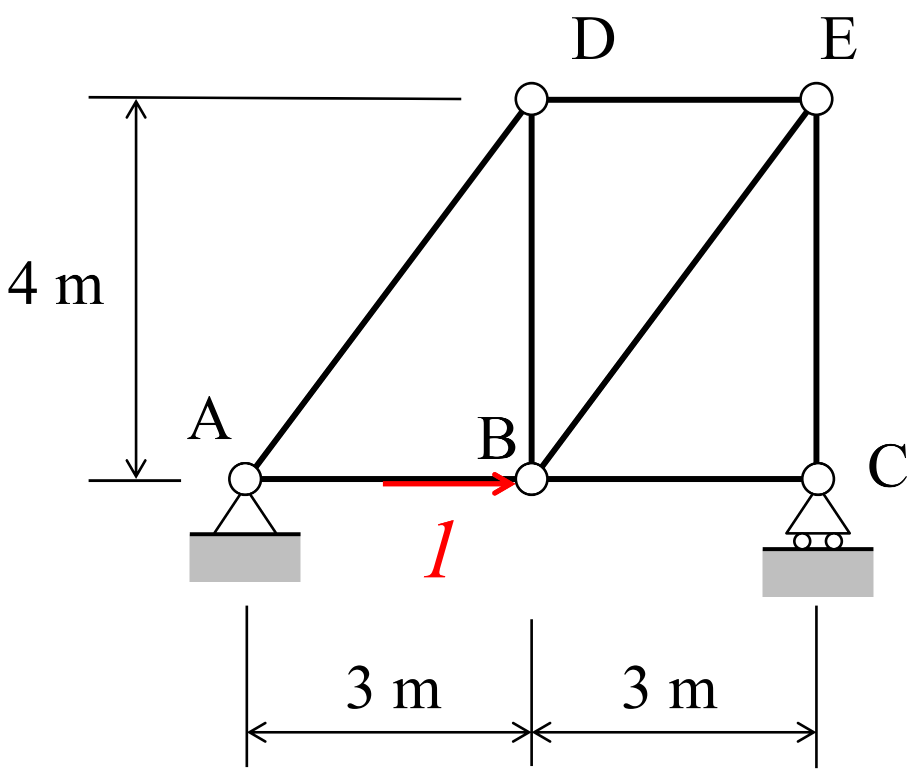
$$1 \cdot \delta_{Bv} = \sum_{i=1}^{7} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i} = -0.1086 \text{ cm}$$



Negative sign indicates that deflection is in the opposite direction of the virtual force

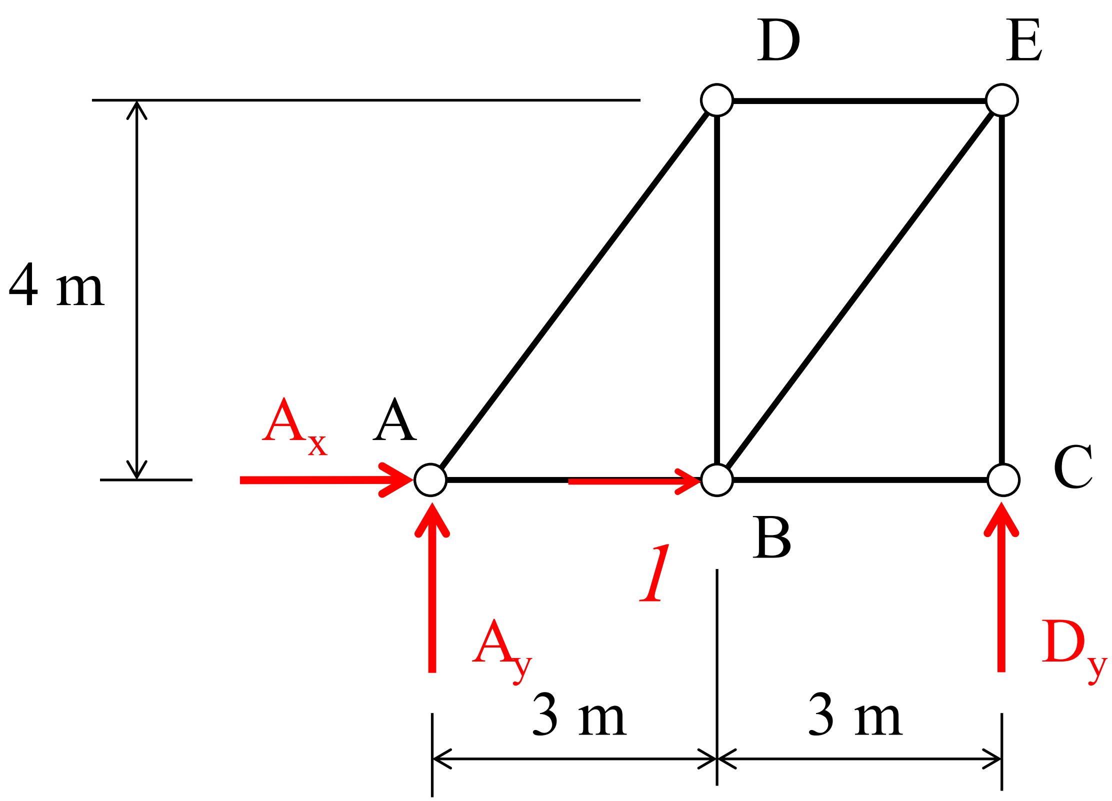
$$\delta_{Bv} = 1.086 \text{ cm downward}$$

Virtual System to Measure δ_{Bh}



1. Remove all loads from the structure;
2. Apply a unit, dimensionless virtual load **in-line** with the real displacement, δ_{Bv}, that we want to find;
3. Perform a truss analysis to find all truss member virtual axial forces, F_{Qi}

Find Support Reactions



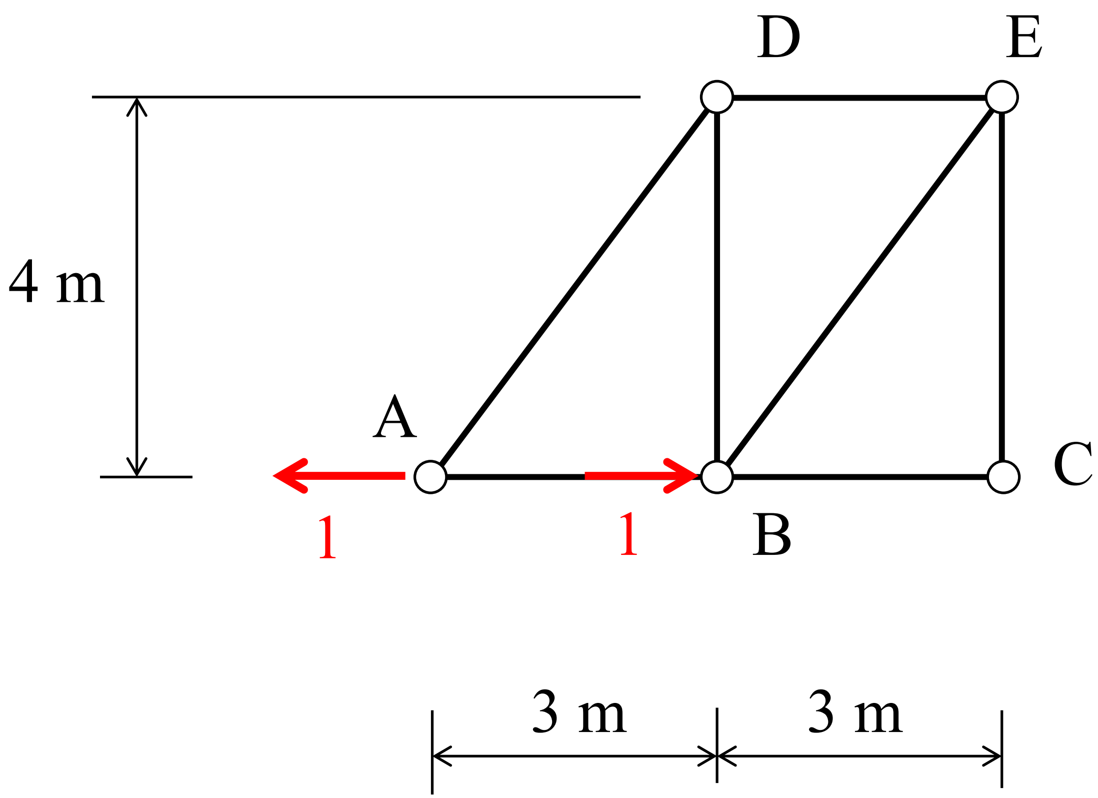
$$\circlearrowleft + \sum M_A = 0$$

$$+\uparrow \sum F_y = 0$$

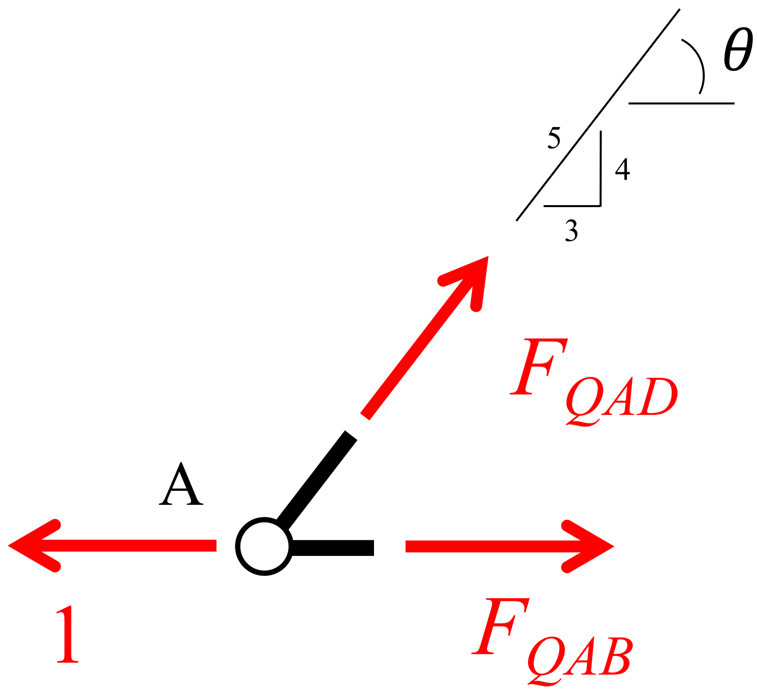
$$\xrightarrow{+} \sum F_x = 0$$

$A_x = -1$

Virtual System Support Reactions



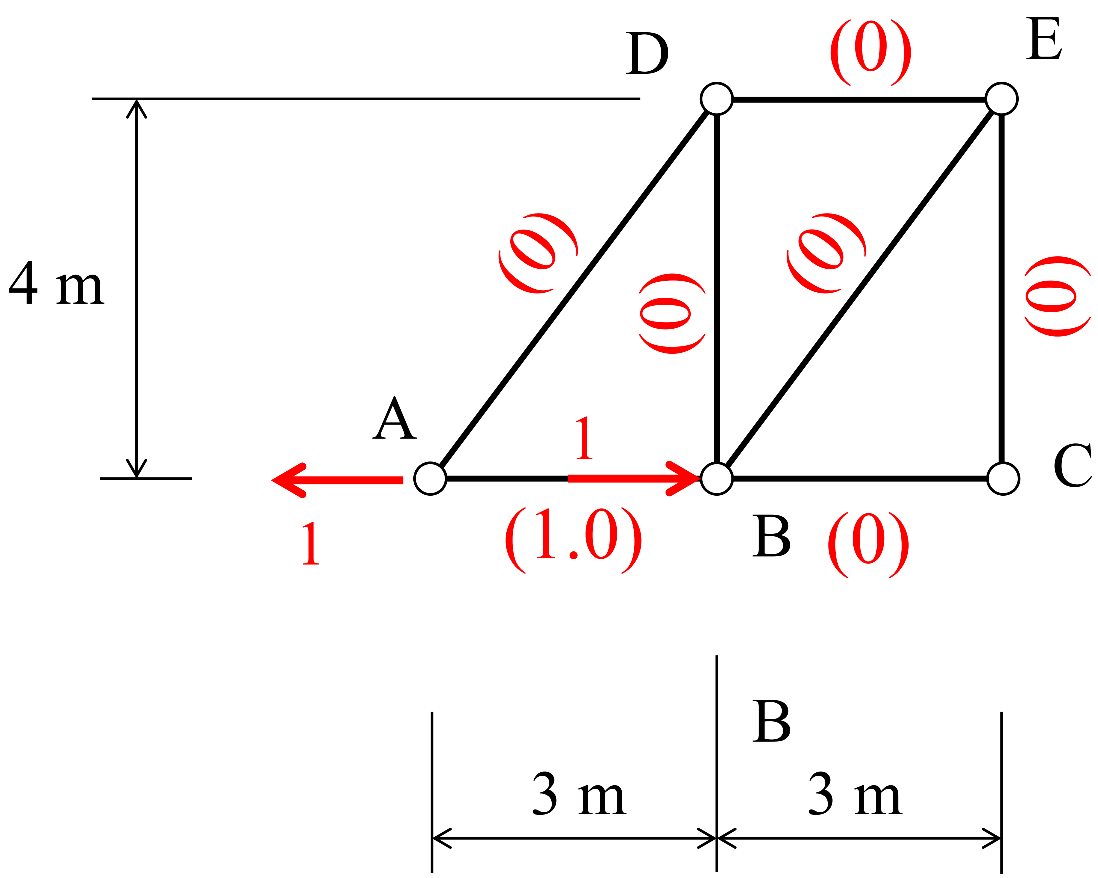
FBD of Joint A



$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

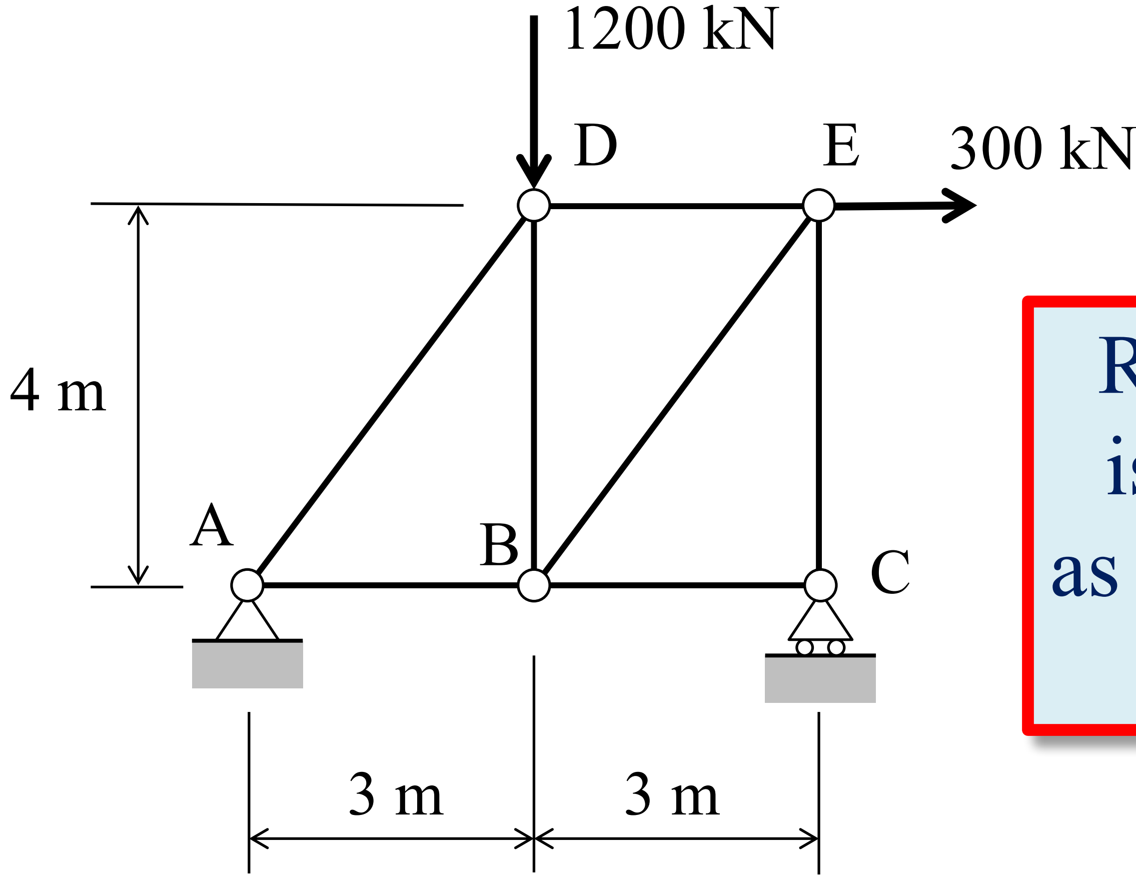
Show Results on FBD of Entire Truss



Virtual truss member forces, F_{Qi}

Tension is Positive

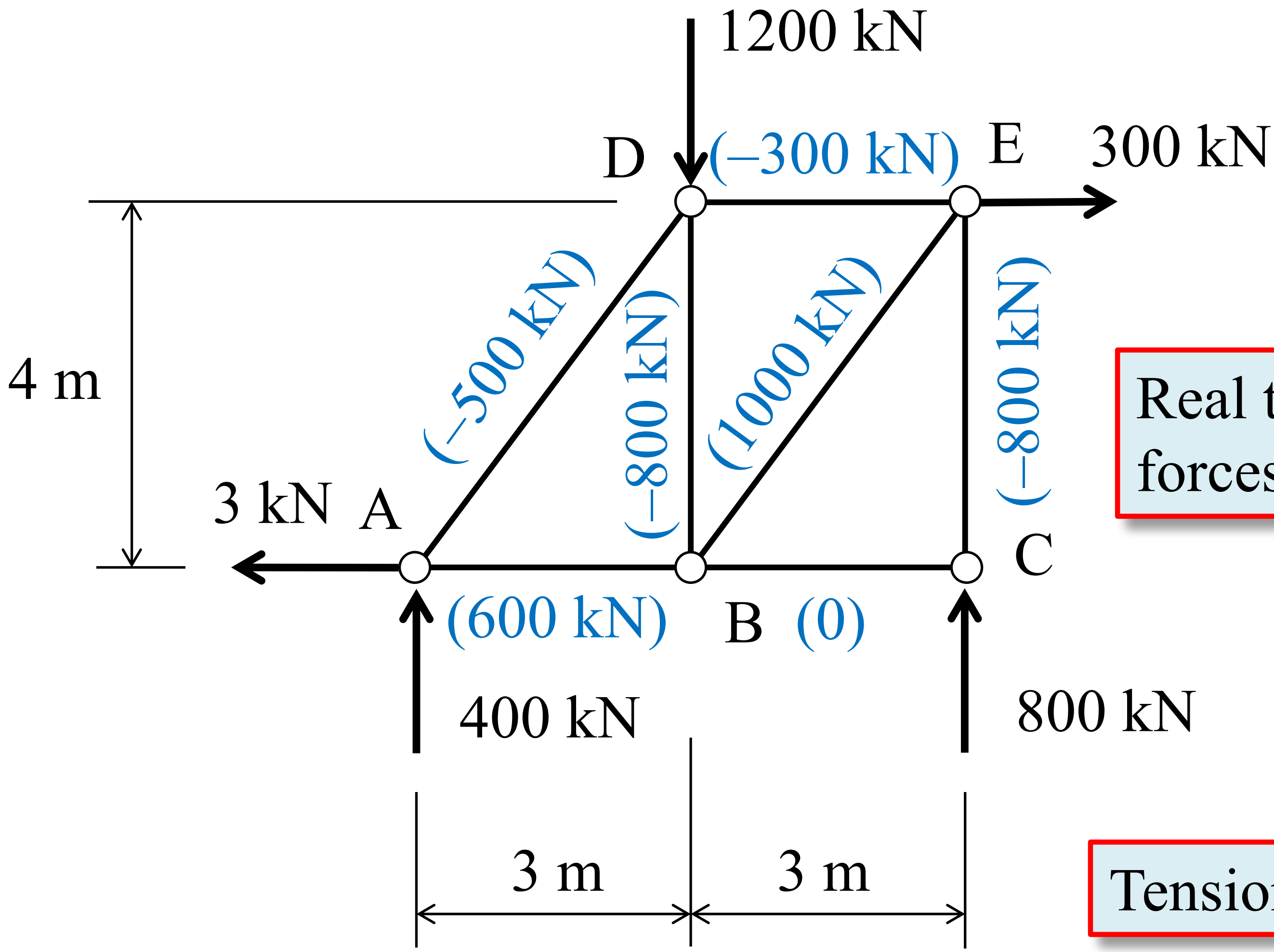
Step 2 –Real Analysis



Real system is the same as the previous analysis

1. Place all of the loads on the structure;
2. Perform a truss analysis to find all truss member real axial forces, F_{Pi}

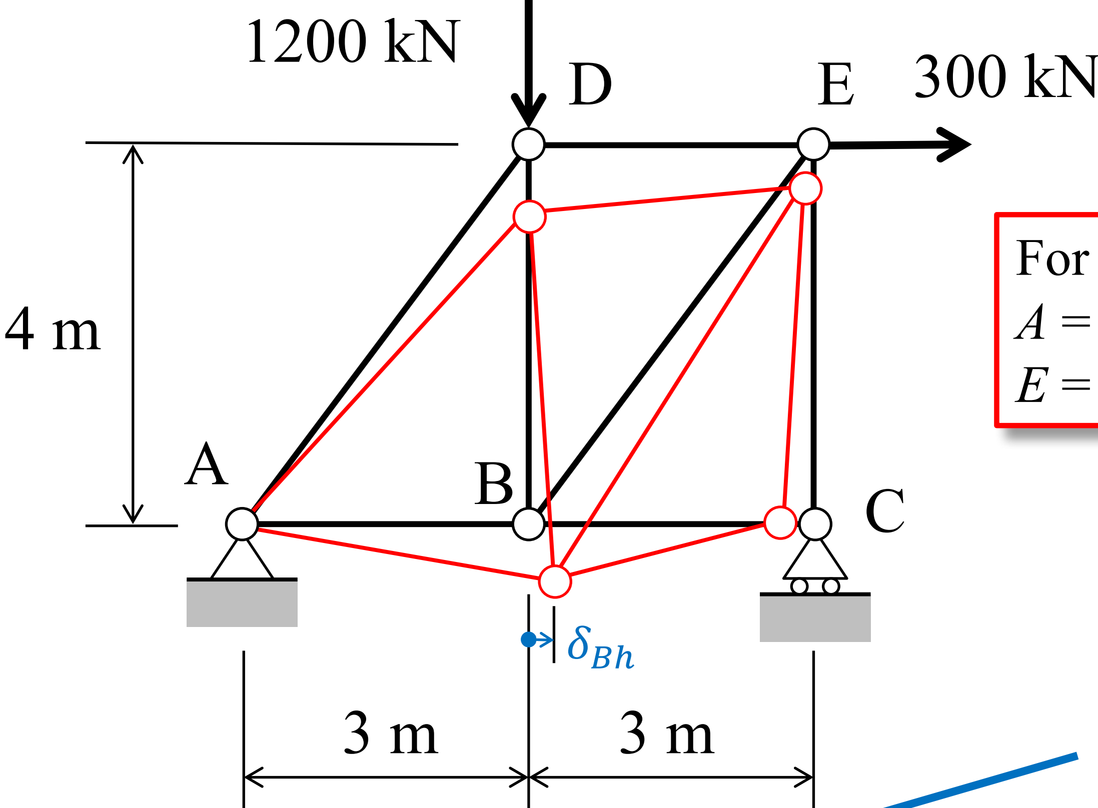
Show Results on FBD of Entire Truss



Real truss member forces, F_{Pi}

Tension is Positive

Step 3 – Use the Principle of Virtual Work to Find δ_{Bh}



For all truss members use:
$A = 25\ \text{cm}^2$
$E = 210\ \text{GPa}$

$$1 \cdot \delta_{Bh} = \sum_{i=1}^{n} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i}$$

From Step 2 – real analysis

From Step 1 –virtual analysis

Use a Table to Organize Virtual Work Calculations

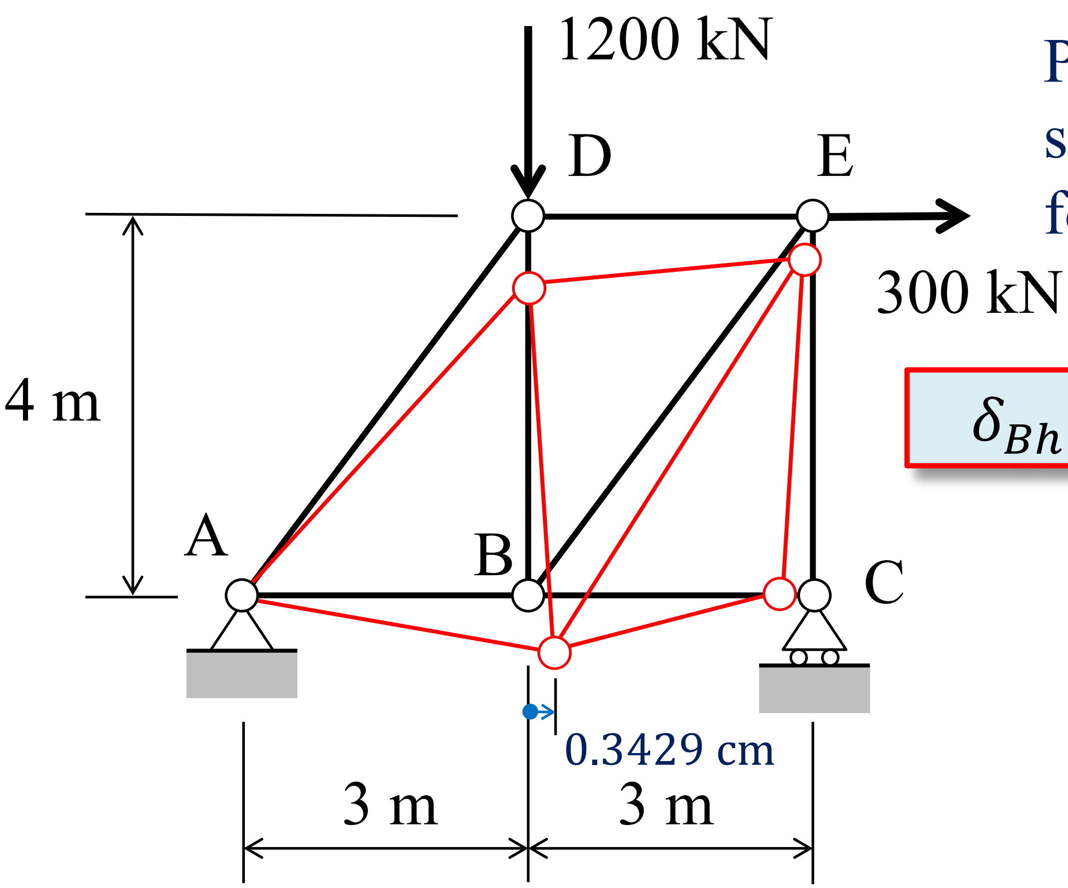
Member	*A (cm²)*	*E (GPa)*	*L (m)*	F_Q	F_P*(kN)*	U_Q*(cm)*
AD	25	210	5	0	– 500	0
AB	25	210	3	1.0	600	0.3429
BD	25	210	4	0	– 800	0
DE	25	210	3	0	– 300	0
BE	25	210	5	0	1000	0
BC	25	210	3	0	0	0
EC	25	210	4	0	– 800	0
Total						**0.3429**

Sample Calculation

$$F_{QAB} \frac{F_{PAB} L_{AB}}{A_{AB} E_{AB}} = 1.0 \left[\frac{(600 \text{ kN})(3 \text{ m}) \left(\frac{100 \text{ cm}}{\text{m}}\right)}{(25 \text{ cm}^2)(210 \text{ kN/mm}^2) \left(\frac{100 \text{ mm}^2}{\text{cm}^2}\right)} \right] = 0.3429 \text{ cm}$$

Results for δ_{Bh}

$$1 \cdot \delta_{Bh} = \sum_{i=1}^{7} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i} = 0.3429 \text{ cm}$$



Positive so deflection is in the same direction of the virtual force

$$\delta_{Bh} = 0.3429 \text{ cm to the right}$$

Results for Deflection at Point B