Method of Virtual Work for Trusses

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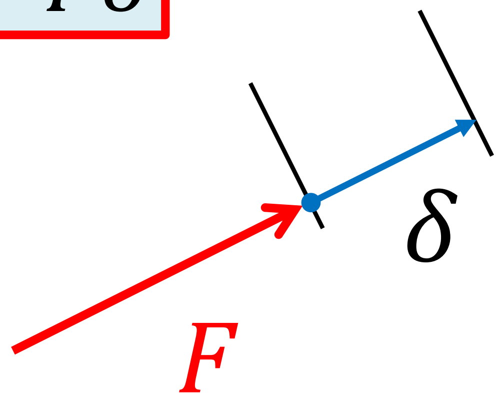
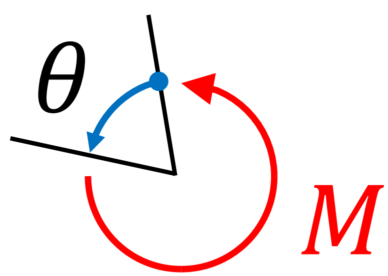
Work Done by Force/Moment

$$W = F\delta$$



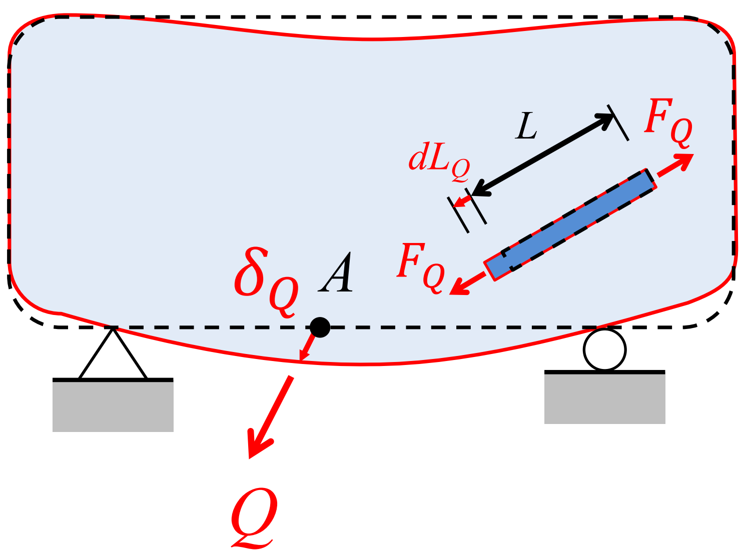
Work is done by a force acting through and in-line displacement

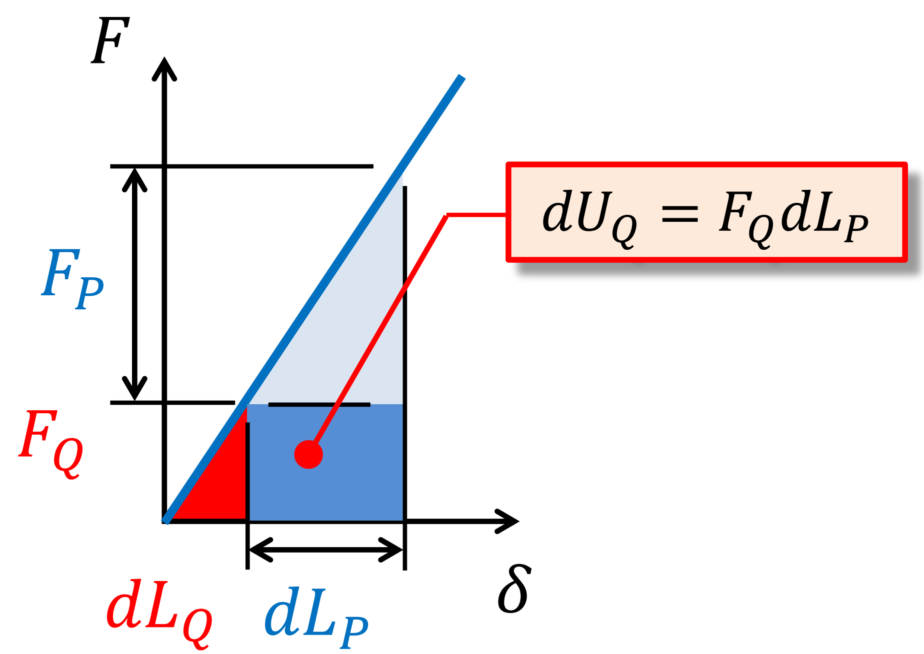
$$W = M\theta$$

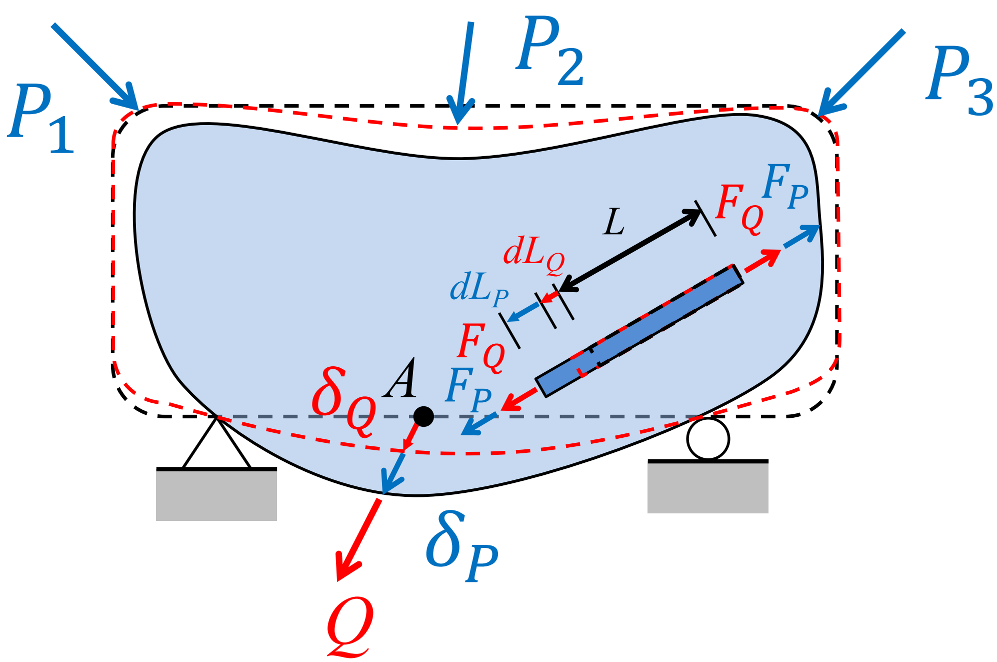


Work is done by a moment acting through and in-line rotation

Virtual Work for a General Body



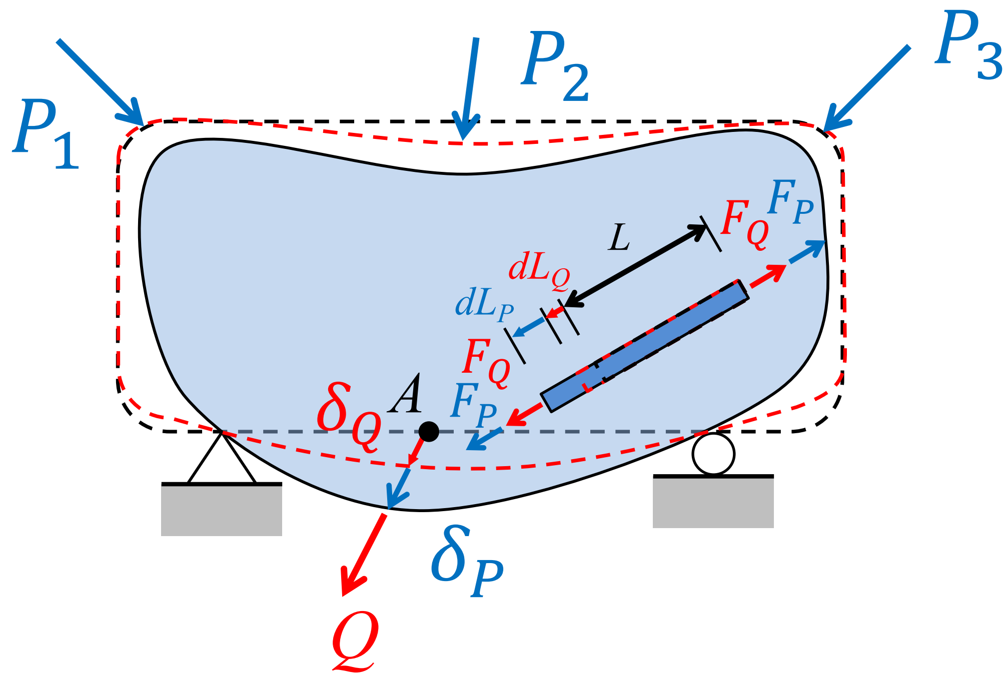




$$U_Q = \iint F_Q dL_P$$

$$W_Q = Q\delta_P$$

Virtual Work for a General Body



$$W_Q = U_Q$$

$$U_Q = \iint F_Q dL_P$$

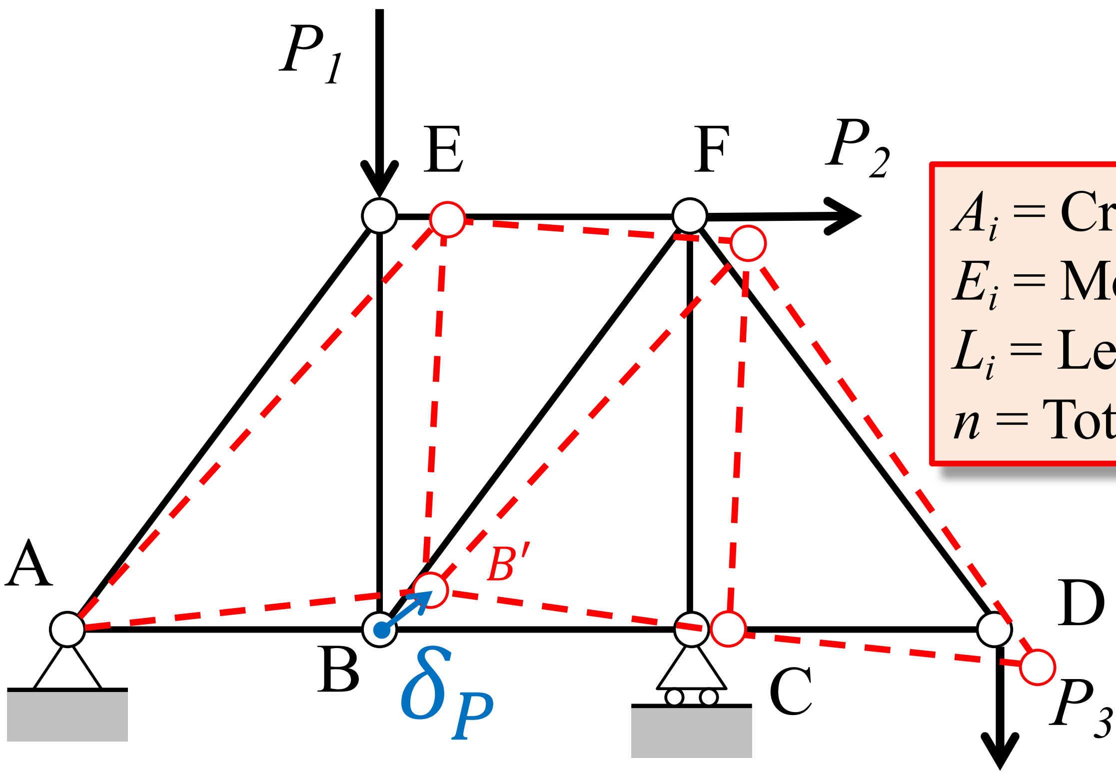
$$W_Q = Q\delta_P$$

$$Q\delta_P = \iint F_Q dL_P$$

Real Deformation

Virtual Loads

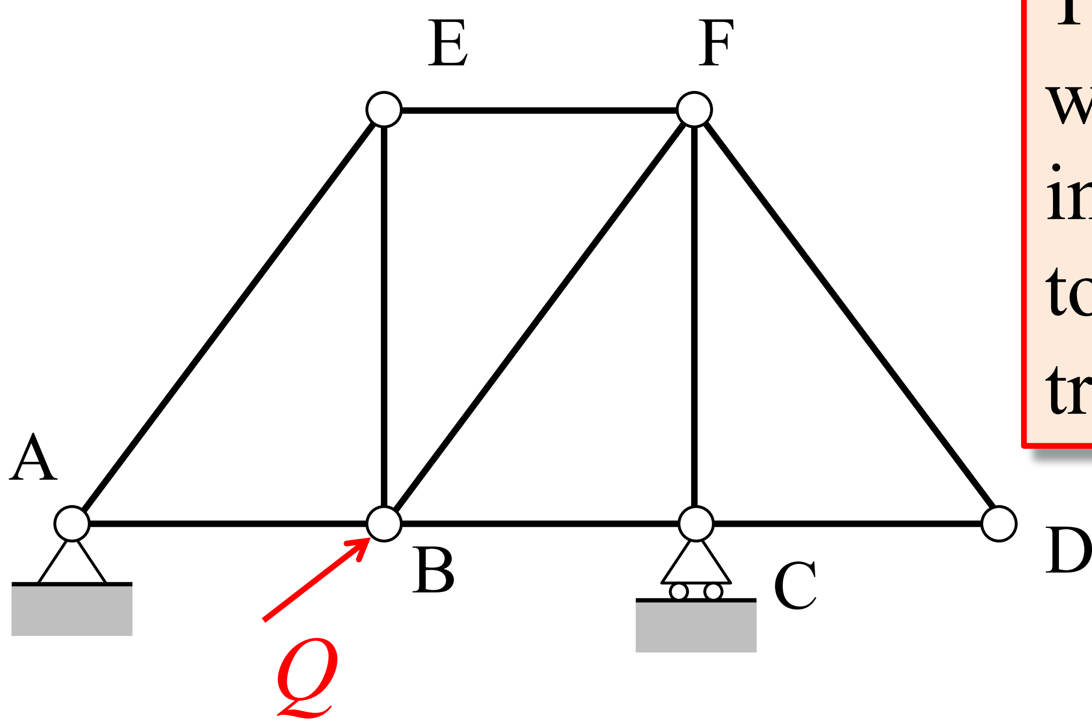
Consider a Truss Structure Subjected To Joint Loads



A_i = Cross sectional area
E_i = Modulus of Elasticity
L_i = Length of truss member
n = Total number of truss members

We want to find the deflection of joint B due to the applied loads

Apply Virtual Force

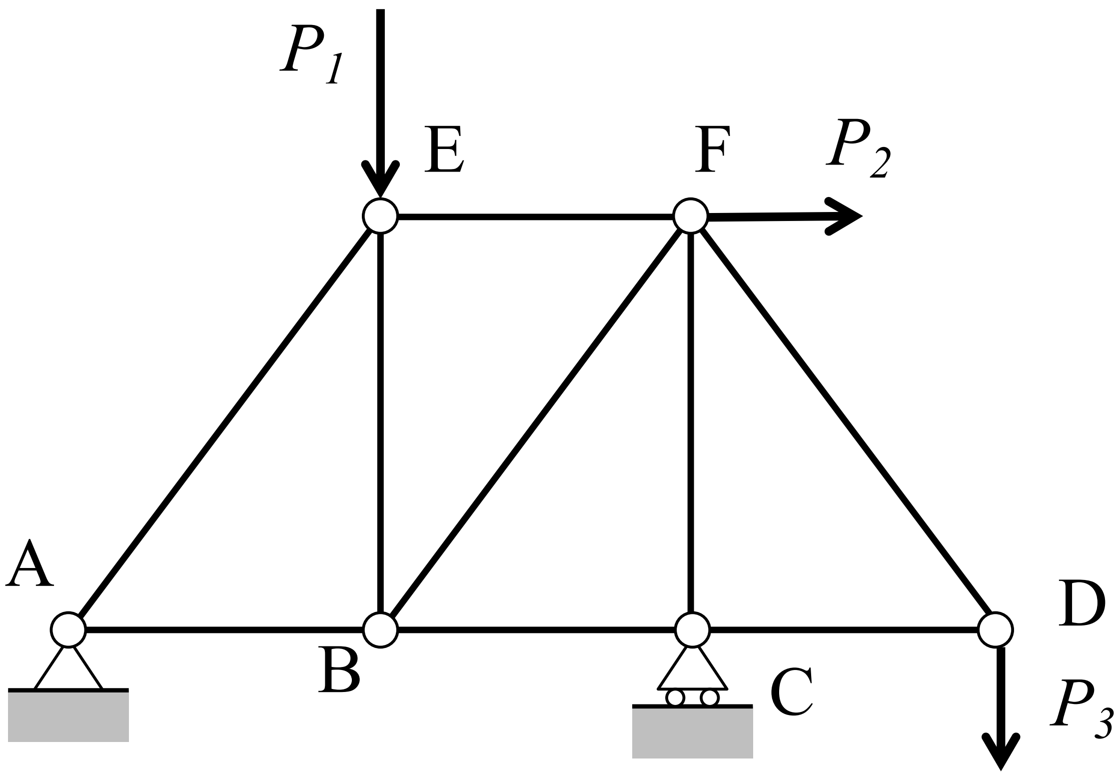


The virtual force will cause an internal axial force to develop in each truss member, F_{Qi}

Apply a virtual force **in-line** with the real displacement δ_P

$$W_Q = Q\delta_P$$

Apply the Real Loads to the Truss



The real loads will cause an internal axial force to develop in each truss member, F_{Pi}

A_i = Cross sectional area
E_i = Modulus of Elasticity
L_i = Length of truss member
n = Total number of truss members

The real loads cause an axial deformation of each truss member, $\Delta L_{Pi} = \dfrac{F_{Pi} L_i}{A_i E_i}$

Virtual Strain Energy for the Truss

Recall the general form for the virtual strain energy developed in an individual fiber

$$U_Q = \iint F_Q \, dL_P$$

$$\sum_{i=1}^{n} \qquad F_{Qi} \qquad \Delta L_{Pi}$$

For a truss structure, we can think of the truss members as individual fibers. From the previous slides:

$$\Delta L_{Pi} = \frac{F_{Pi} L_i}{A_i E_i}$$

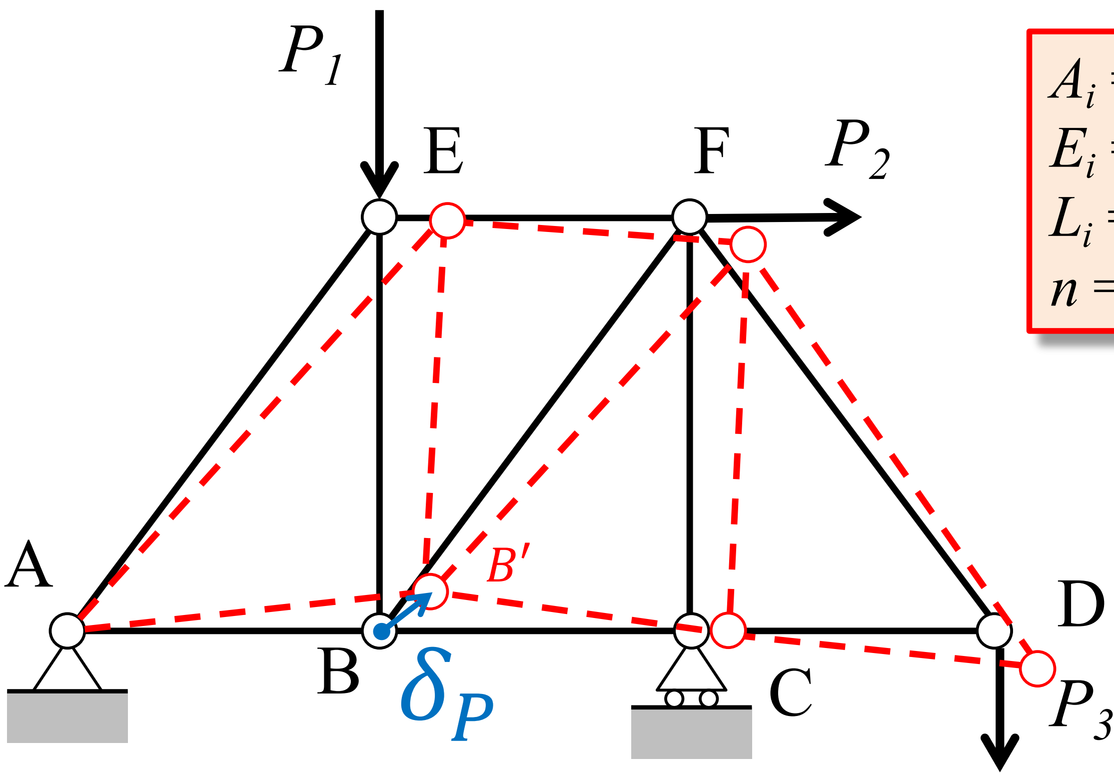
Virtual strain energy developed in an individual truss member *i*

$$U_{Qi} = F_{Qi} \cdot \Delta L_{Pi} = F_{Qi} \frac{F_{Pi} L_i}{A_i E_i}$$

Summing up the virtual strain energy for all of the members, yields the virtual strain energy for the entire truss:

$$U_Q = \sum_{i=1}^{n} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i}$$

Principle of Virtual Work for Truss Deflections



A_i = Cross sectional area
E_i = Modulus of Elasticity
L_i = Length of truss member
n = Total number of truss members

$$W_Q = U_Q$$

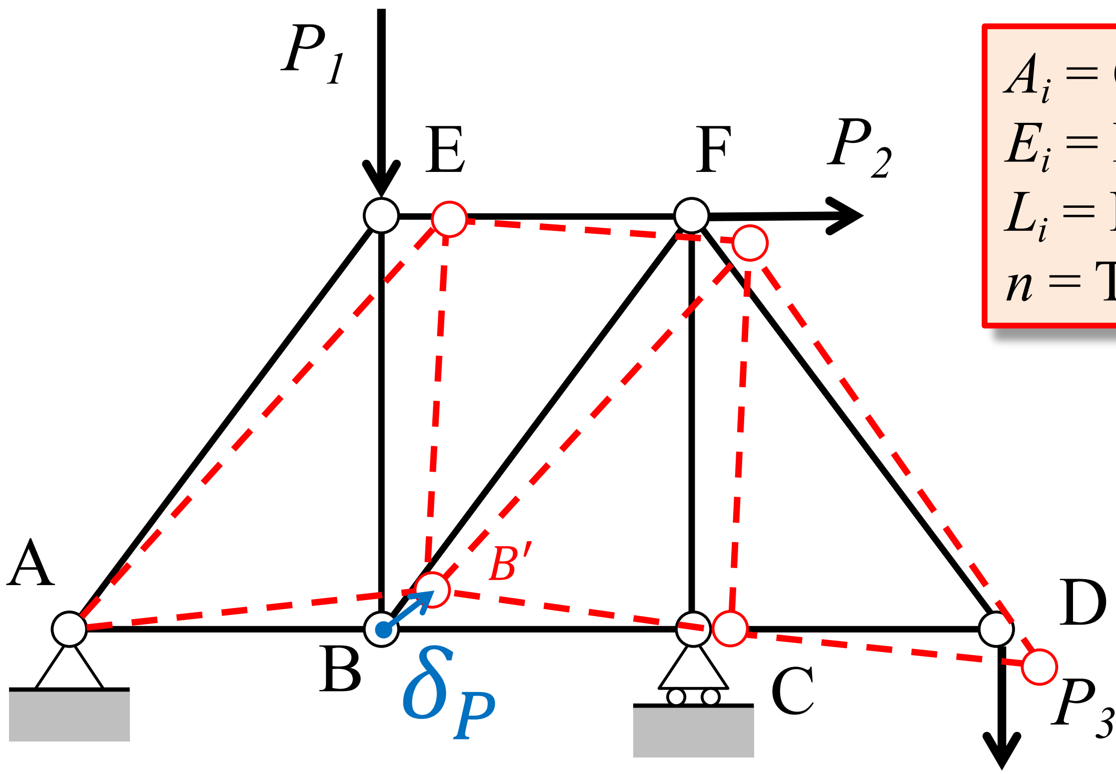
$$Q\delta_P = \iint F_Q dL_P$$

Real Deformation

$$Q\delta_P = \sum_{i=1}^{n} F_{Qi} \frac{F_{Pi}L_i}{A_iE_i}$$

Virtual Loads

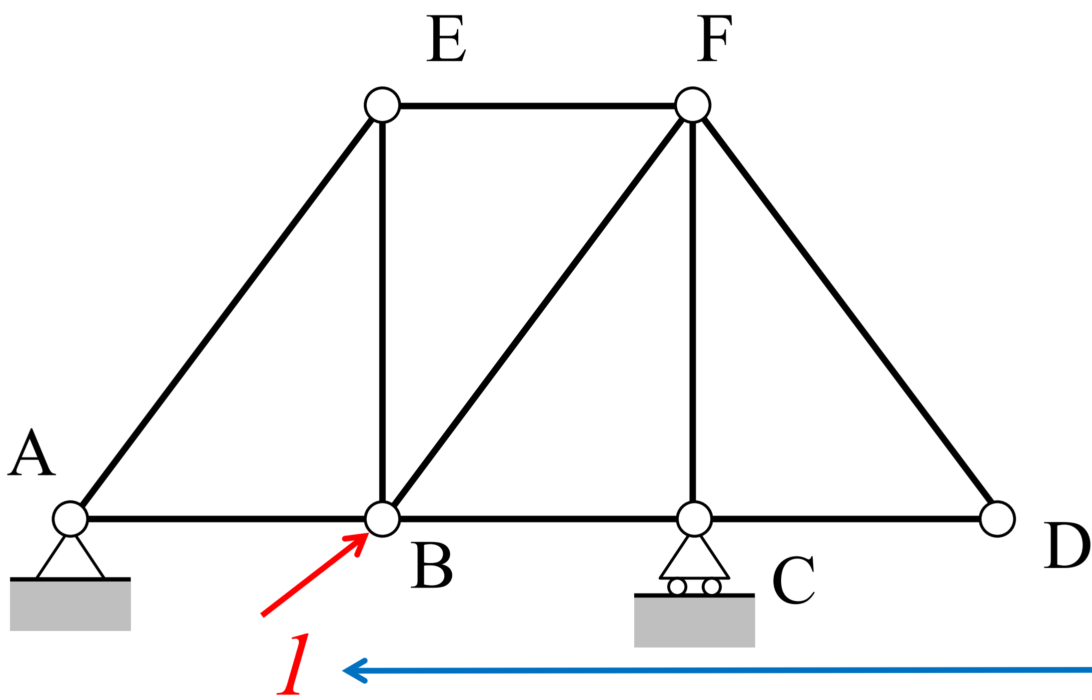
Procedure For Virtual Work Deflection Analysis



A_i = Cross sectional area
E_i = Modulus of Elasticity
L_i = Length of truss member
n = Total number of truss members

We want to find the real deflection of joint B due to the applied loads, δ_P

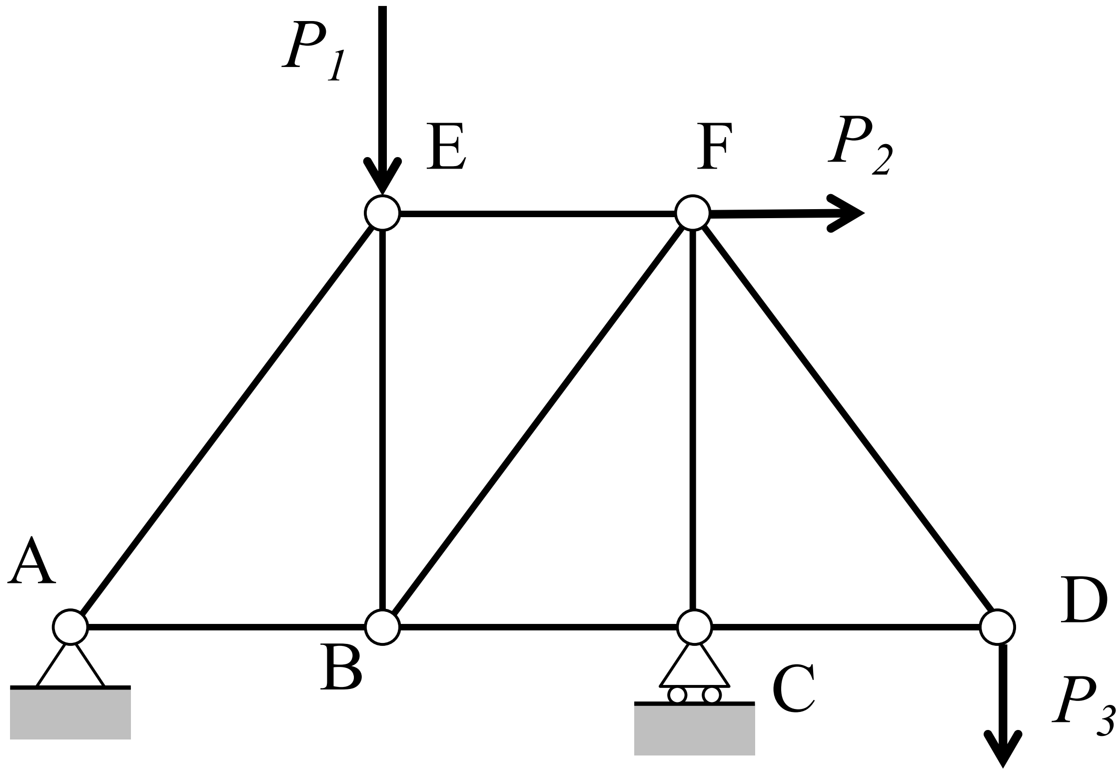
Step 1 –Virtual Analysis



Convenient to set $Q = 1$

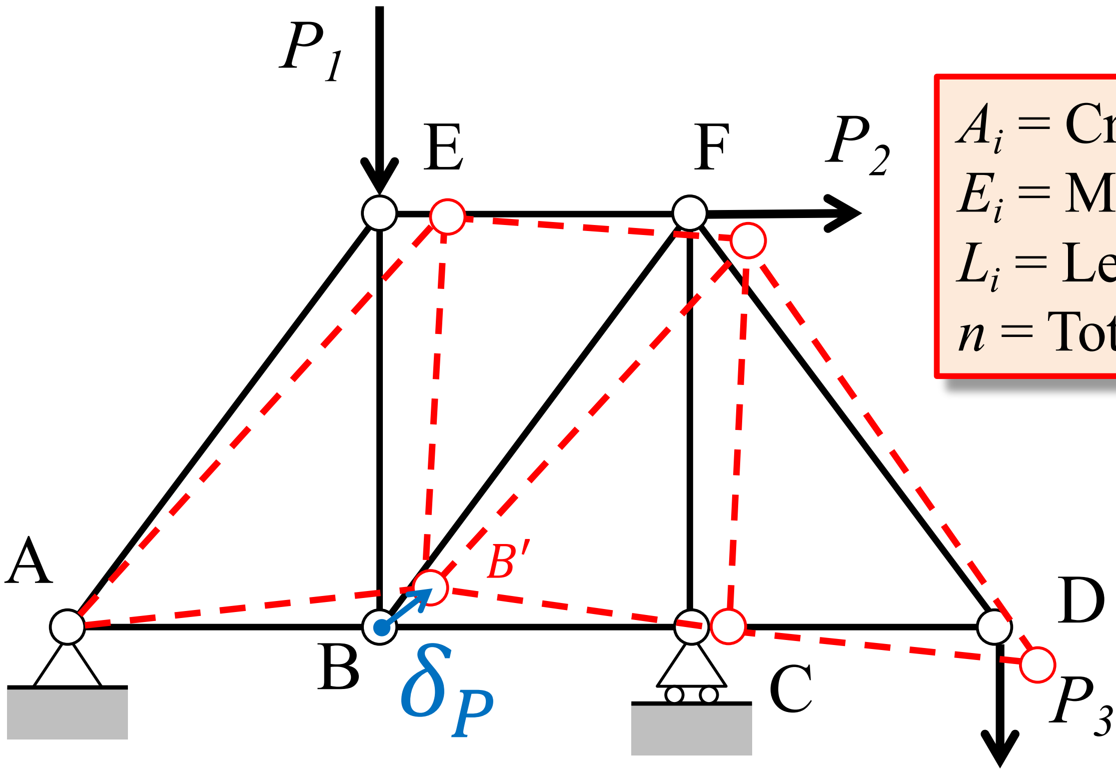
1. Remove all loads from the structure;
2. Apply a unit, dimensionless virtual load **in-line** with the real displacement, δ_P, that we want to find;
3. Perform a truss analysis to find all truss member virtual axial forces, F_{Qi}

Step 2 –Real Analysis



1. Place all of the loads on the structure;
2. Perform a truss analysis to find all truss member real axial forces, F_{Pi}

Step 3 – Use the Principle of Virtual Work to Find δ_P



A_i = Cross sectional area
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L_i = Length of truss member
n = Total number of truss members

$$1 \cdot \delta_P = \sum_{i=1}^{n} F_{Qi} \frac{F_{Pi} L_i}{A_i E_i}$$

From Step 2 – real analysis (F_{Pi})

From Step 1 –virtual analysis (F_{Qi})