Engineering Beam Theory

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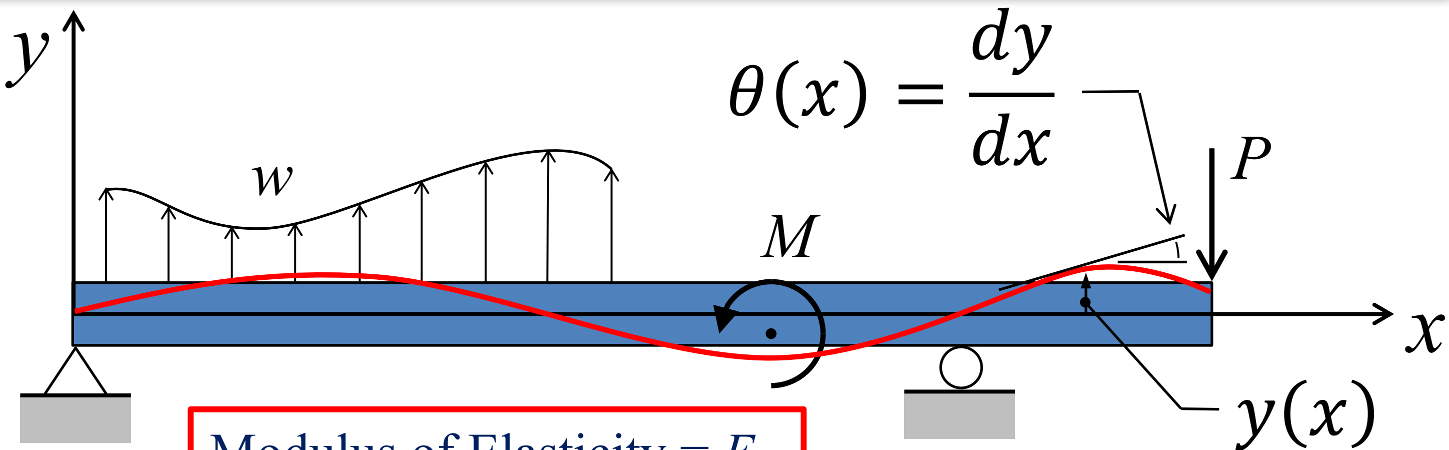
Consider an Elastic Beam with General Supports, and General Loading



Modulus of Elasticity = E
Moment of Inertia = I

We seek *y(x) and θ(x)* that describe the transverse deformation of the neutral axis and the slope of the tangent line to the neutral axis.

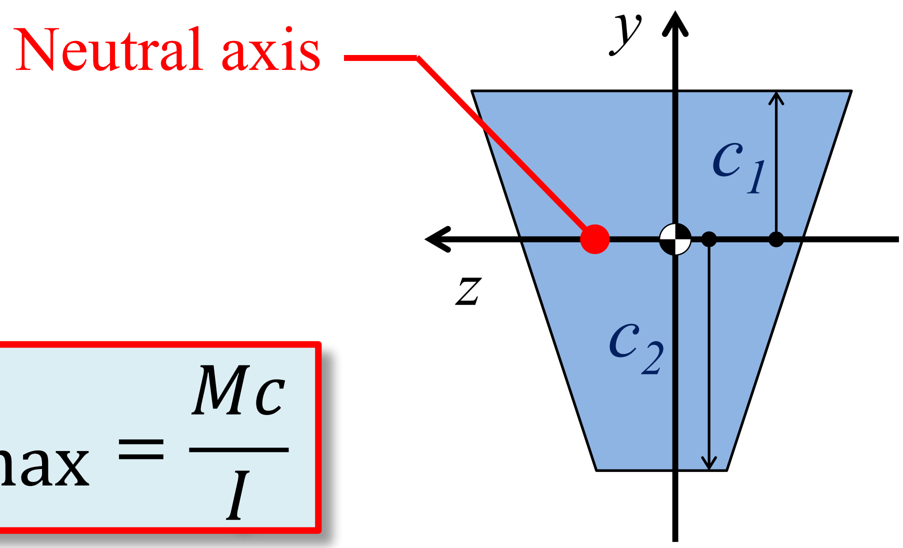
Engineering beam theory assumptions:

- Transverse deformation is small relative to beam span;
- Effect of shear deformation is small so we can use the moment-curvature relationship from pure bending.

Recall Relationships from Pure Bending Analysis

Moment-Curvature relationship

$$\kappa = \frac{M}{EI}$$



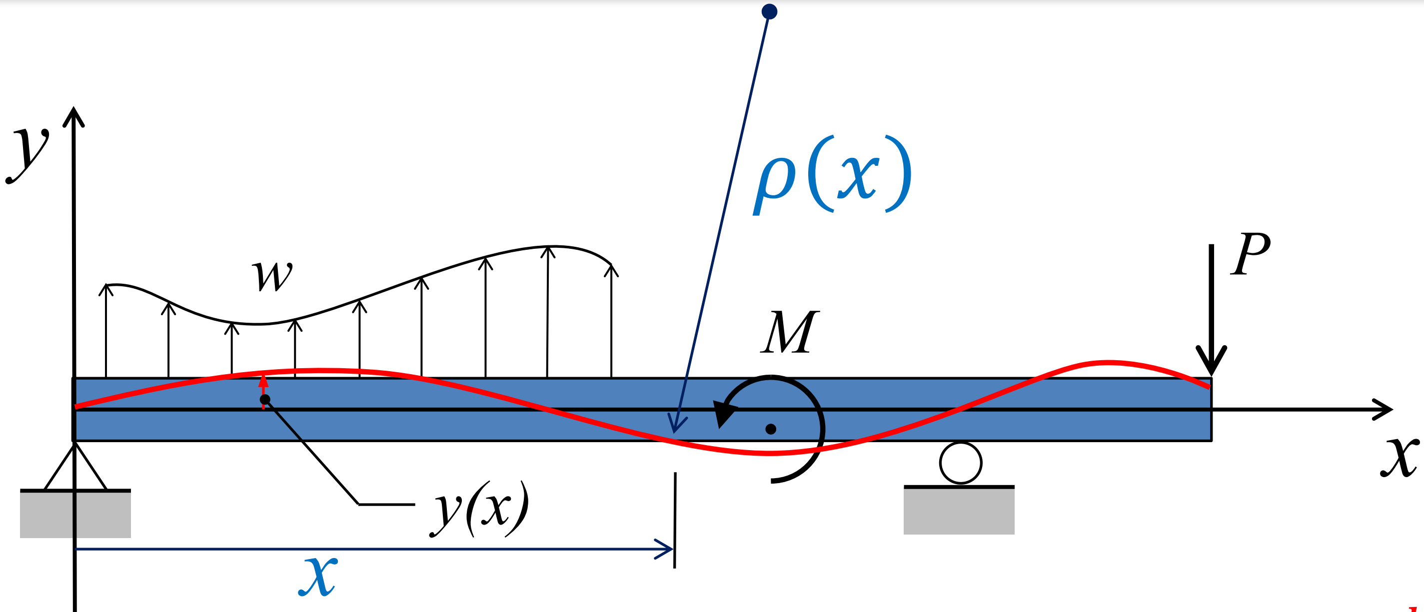
Bending stress distribution

$$\sigma = -\frac{My}{I}$$

$$\sigma_{\max} = \frac{Mc}{I}$$

1. Neutral axis ($\sigma = 0$) is located at the centroid of the beam cross section;
2. Moment-Curvature relationship is basis of bending deformation theory;
3. Bending stress varies linearly over beam cross section and is maximum at the extreme fibers of the beam;

From Analytic Geometry, Recall the Local Curvature of a Function

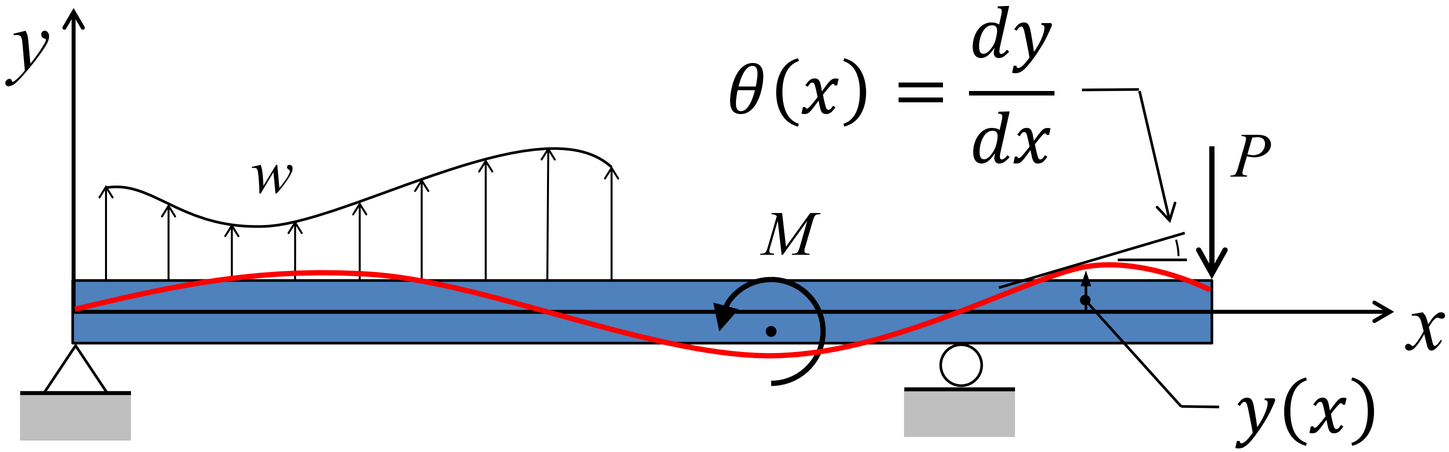


$$\kappa(x) = \frac{1}{\rho(x)} = \frac{\frac{d^2y}{dx^2}}{\left[1 - \left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$

For small deformations: $\frac{dy}{dx} \ll 1$

$$\kappa(x) = \frac{1}{\rho(x)} \approx \frac{d^2y}{dx^2}$$

Moment-Curvature Relationship for Small Deformations



$$\theta(x) = \frac{dy}{dx}$$

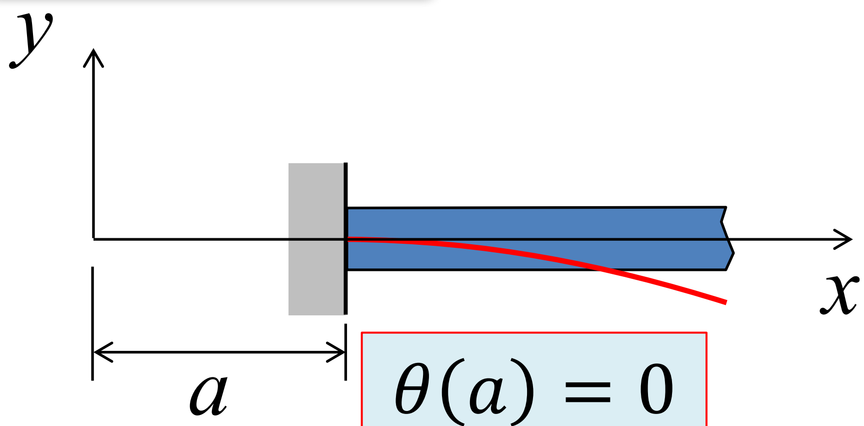
$$\kappa = \frac{d^2y}{dx^2} = \frac{M}{EI}$$

In order to solve this differential equation for y and θ we need:

- Moment equation (from statics);
- Two boundary (or continuity) conditions on y or θ;
- Information on E and I.

Common Boundary Conditions for Beam Problems

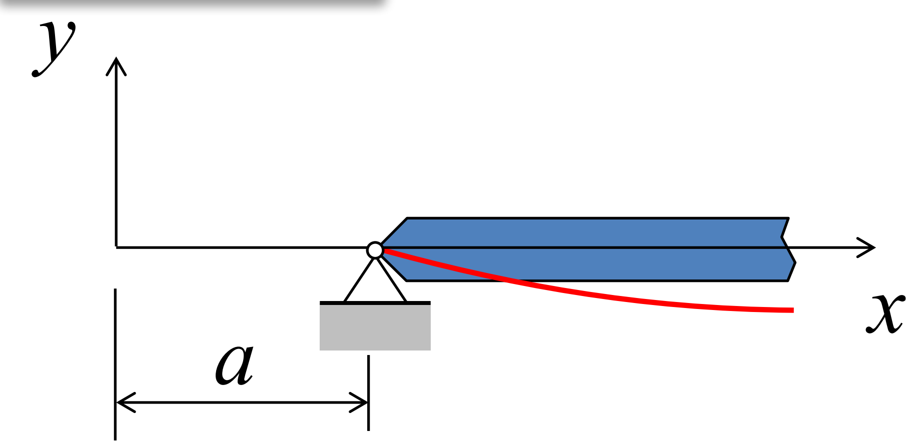
Fixed Support



$$\theta(a) = 0$$

$$y(a) = 0$$

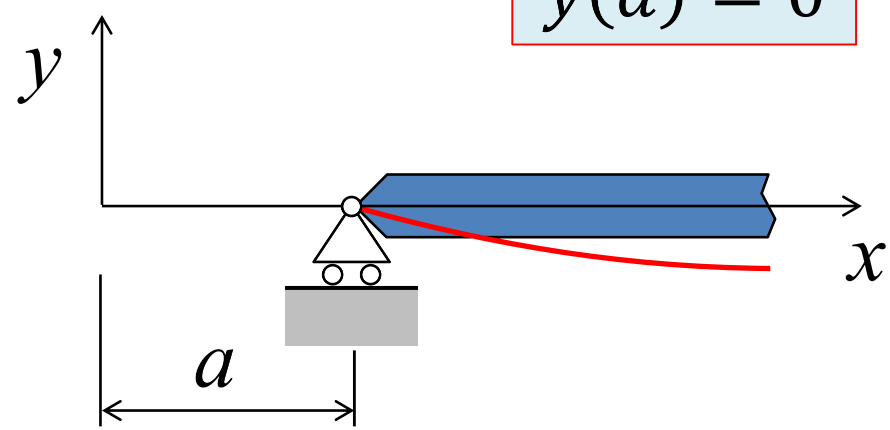
Pin Support



$$\theta(a) = ?$$

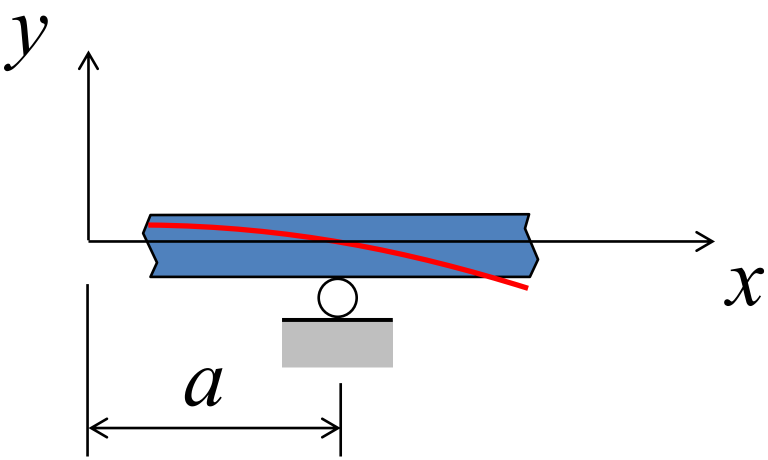
$$y(a) = 0$$

Roller Support



Common Continuity Conditions for Beam Problems

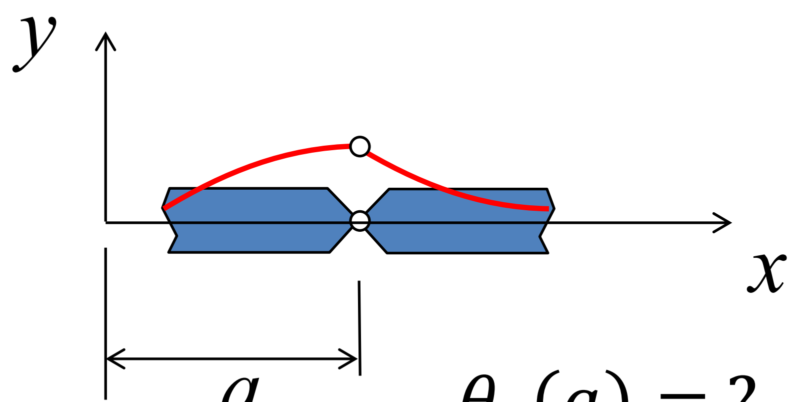
Internal Support



$$\theta_L(a) = \theta_R(a)$$

$$y_L(a) = y_R(a) = 0$$

Internal Hinge



$$\theta_L(a) = ?$$

$$\theta_R(a) = ?$$

$$y_L(a) = y_R(a)$$