Relationships between Shear Force, Bending Moment, and Distributed Loads

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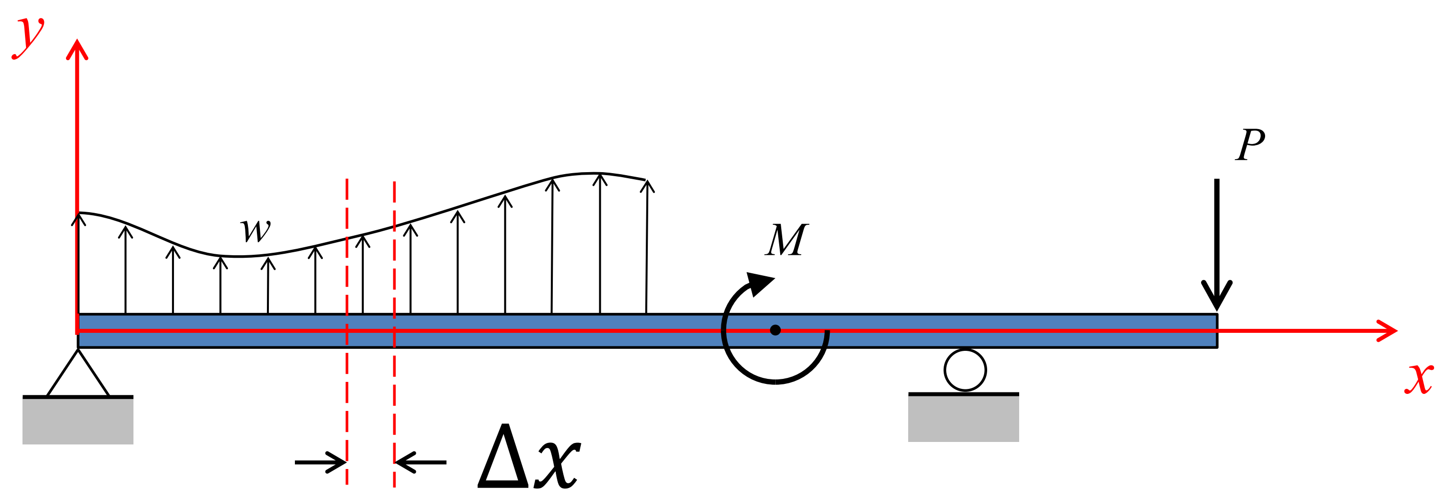
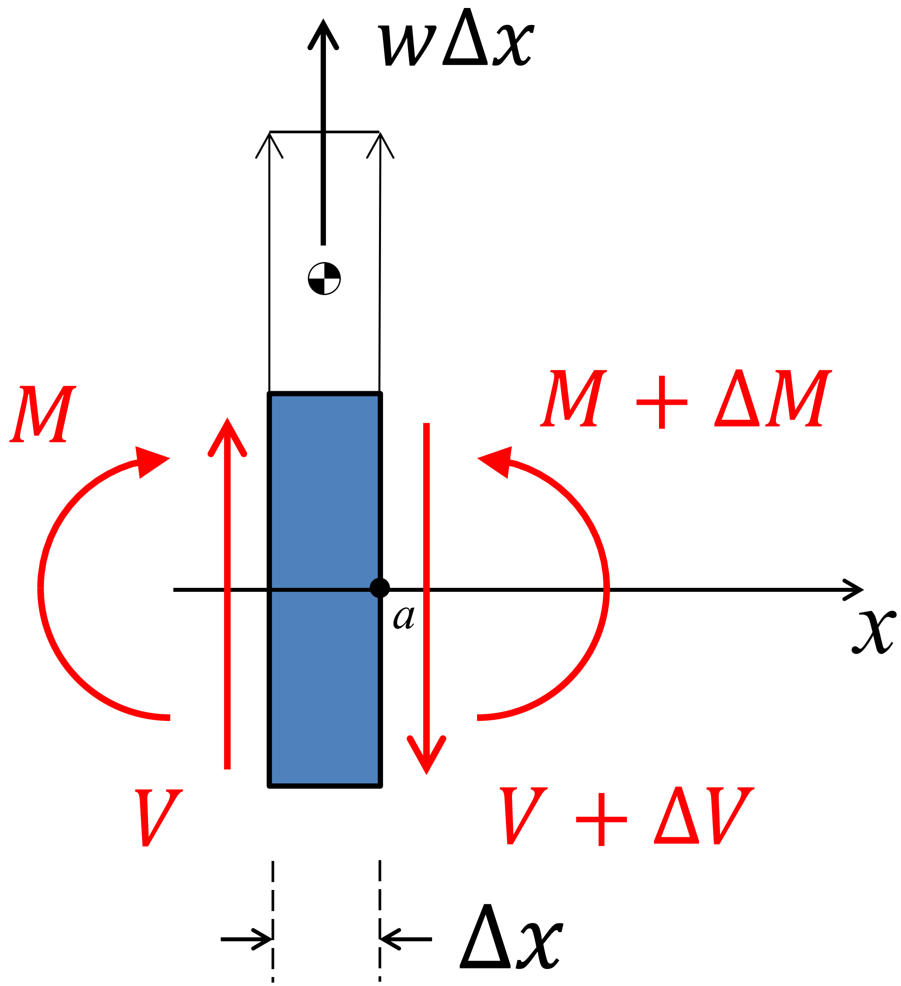
Consider a beam with general supports, and general loading

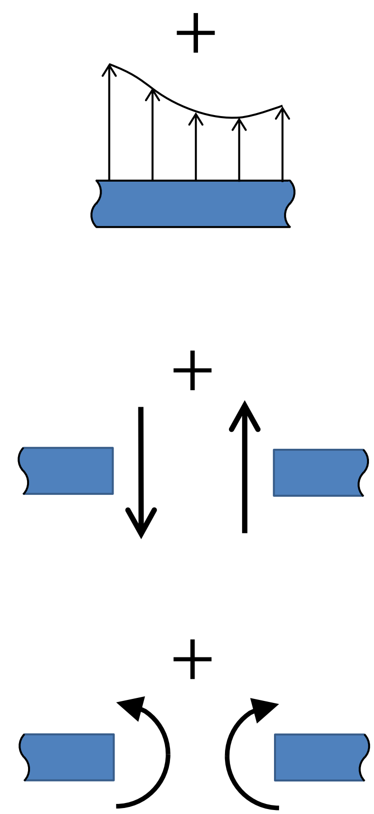


Look at equilibrium of a small slice of the beam

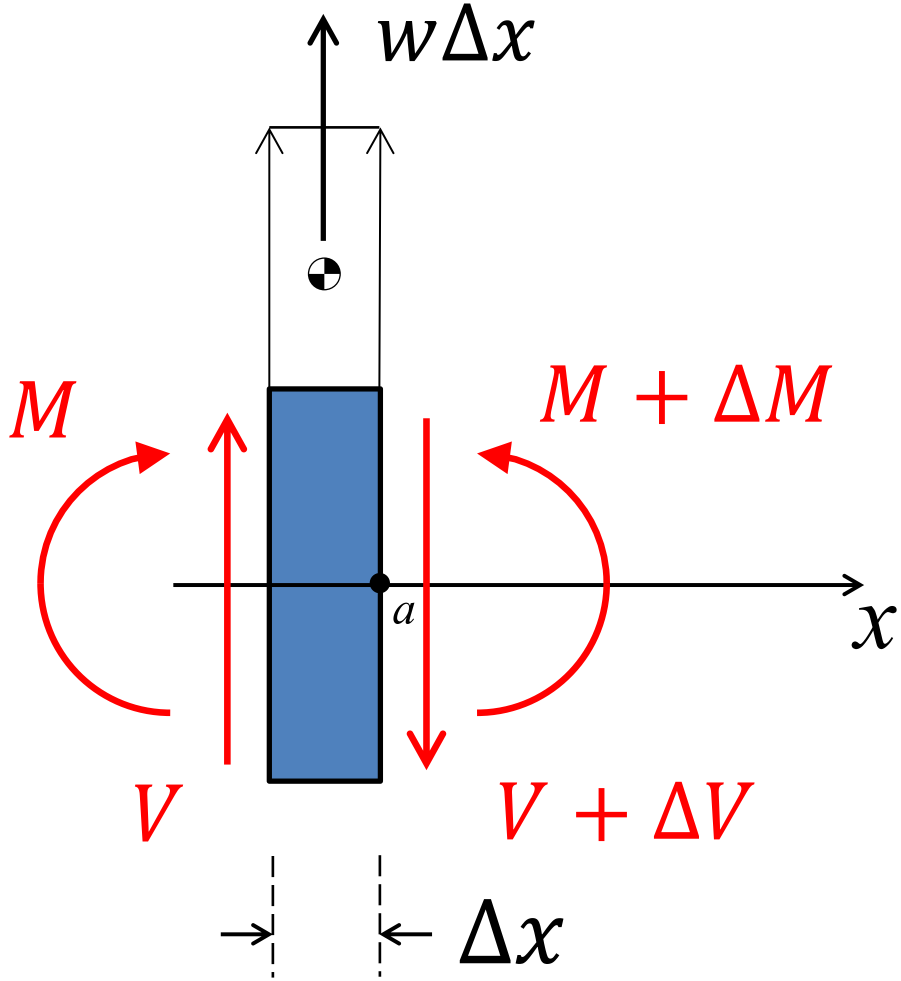
FBD of slice of beam



Sign convention



Force Equilibrium



$$+\uparrow \sum F_y = 0$$

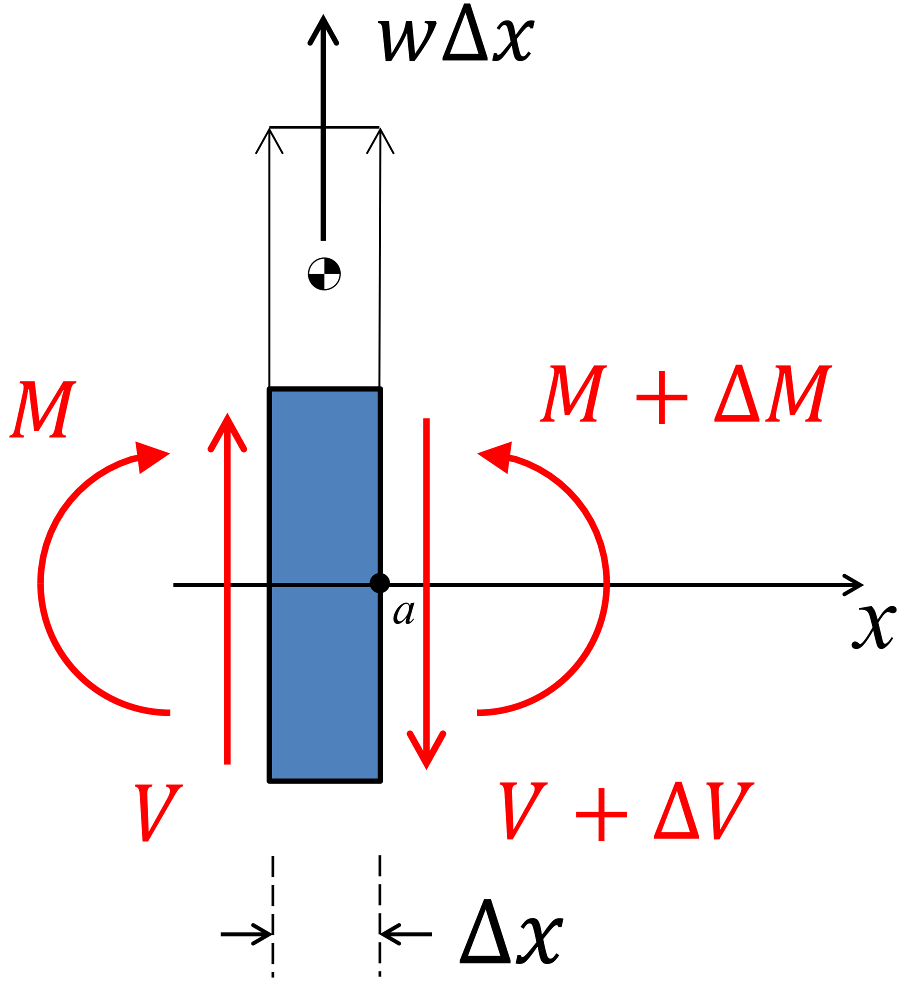
$$\frac{dV}{dx} = w$$

Differential Relationship between V and w

$$\frac{dV}{dx} = w$$

Slope of tangent line to the shear diagram at a point	=	Value of the magnitude of the distributed load intensity at that point

Moment Equilibrium



$$\circlearrowleft + \sum M_a = 0$$

$$\frac{dM}{dx} = V$$

Differential Relationship between M and V

$$\frac{dM}{dx} = V$$

Slope of tangent line to the moment diagram at a point = Value of the magnitude of the ordinate of the shear diagram at that point

Can integrate the differential relationship between w and V between two points on the beam

$$\frac{dV}{dx} = w$$

$$dV = wdx$$

$$\int_A^B dV = \int_A^B wdx$$

$$V_B - V_A = \int_A^B wdx =$$ area under the distributed load between points A and B

Can integrate the differential relationship between V and M between two points on the beam

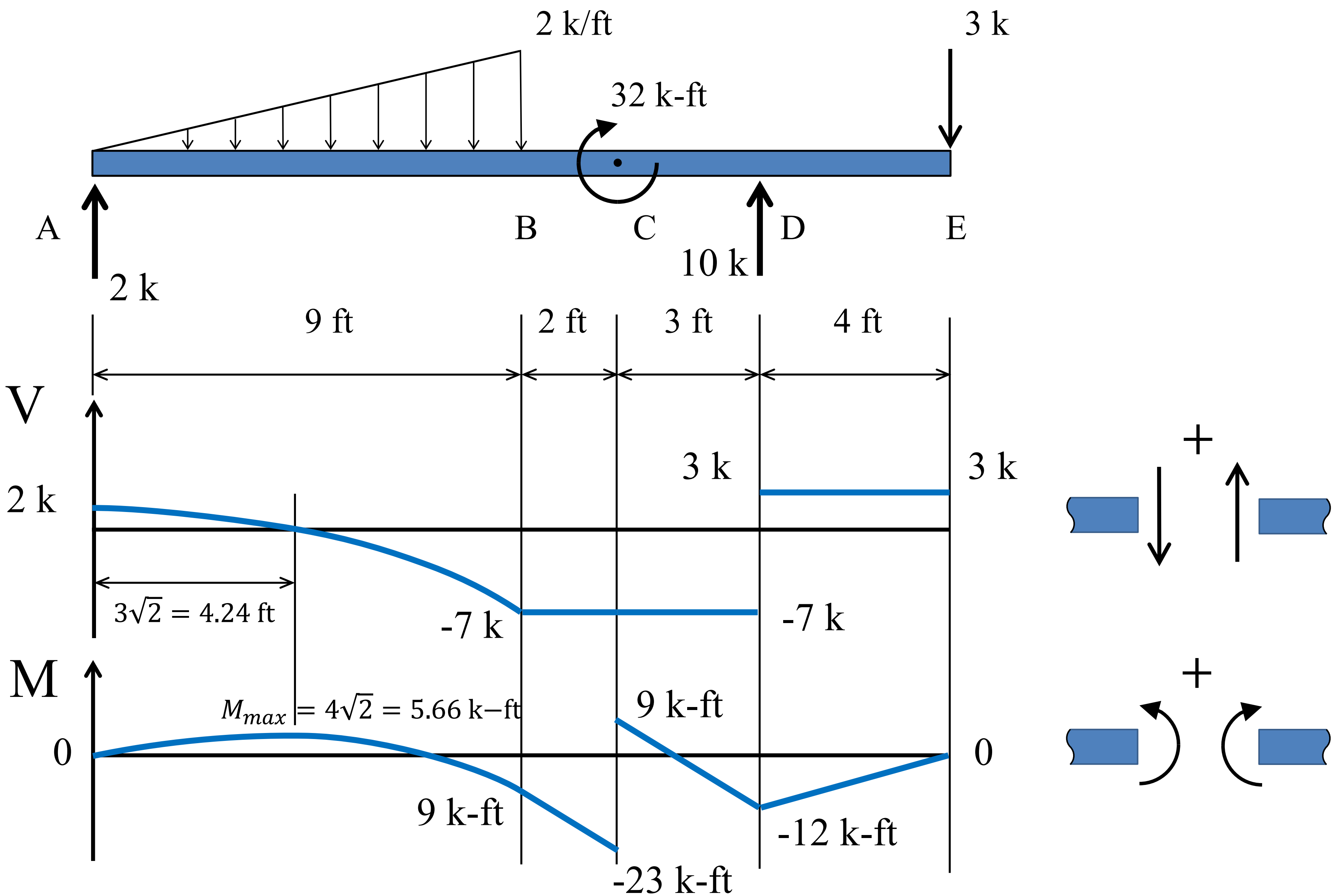
$$\frac{dM}{dx} = V$$

$$dM = V dx$$

$$\int_A^B dM = \int_A^B V dx$$

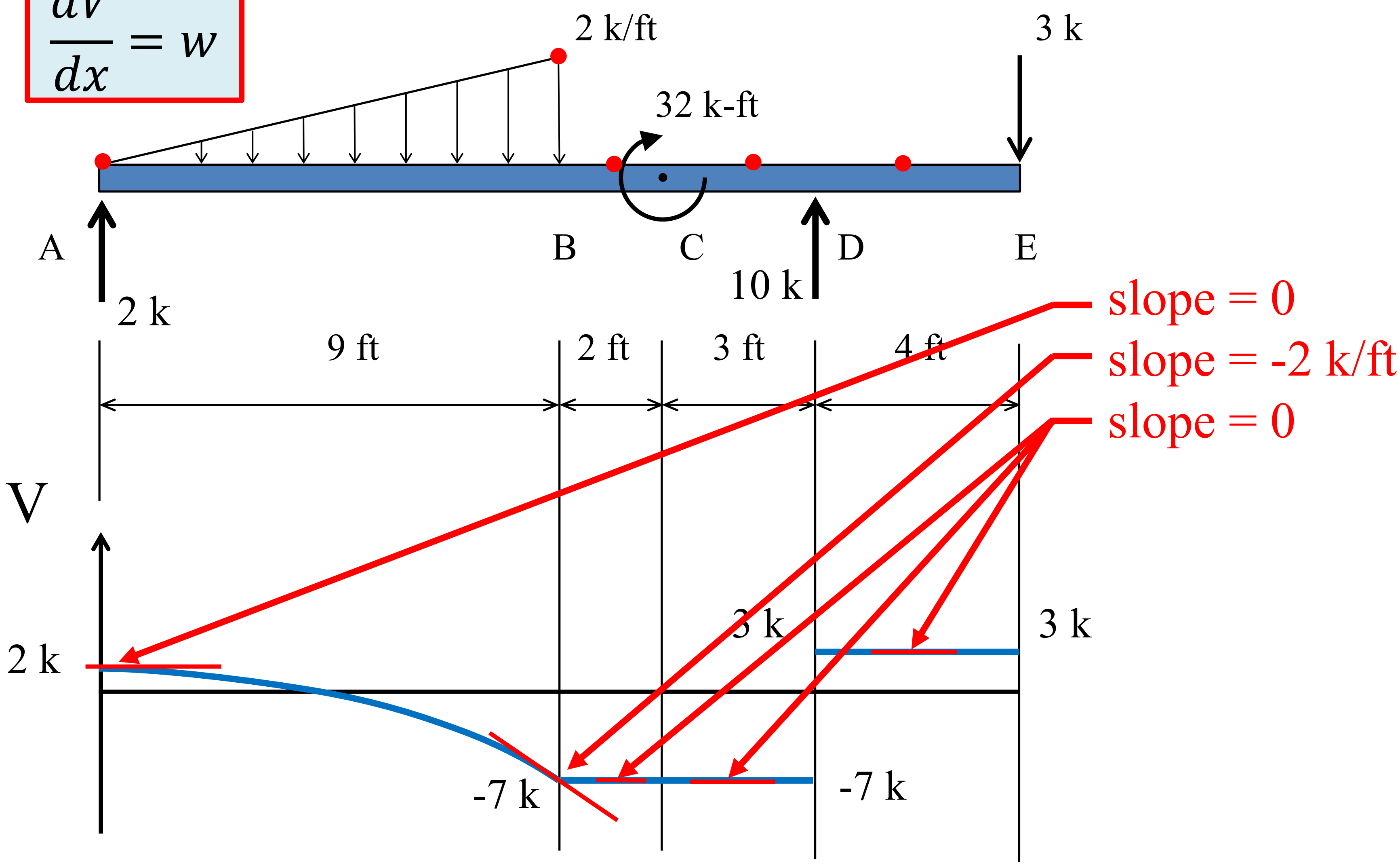
$M_B - M_A = \int_A^B V dx =$ area under the shear diagram between points A and B

Consider the V and M diagrams for the beam below

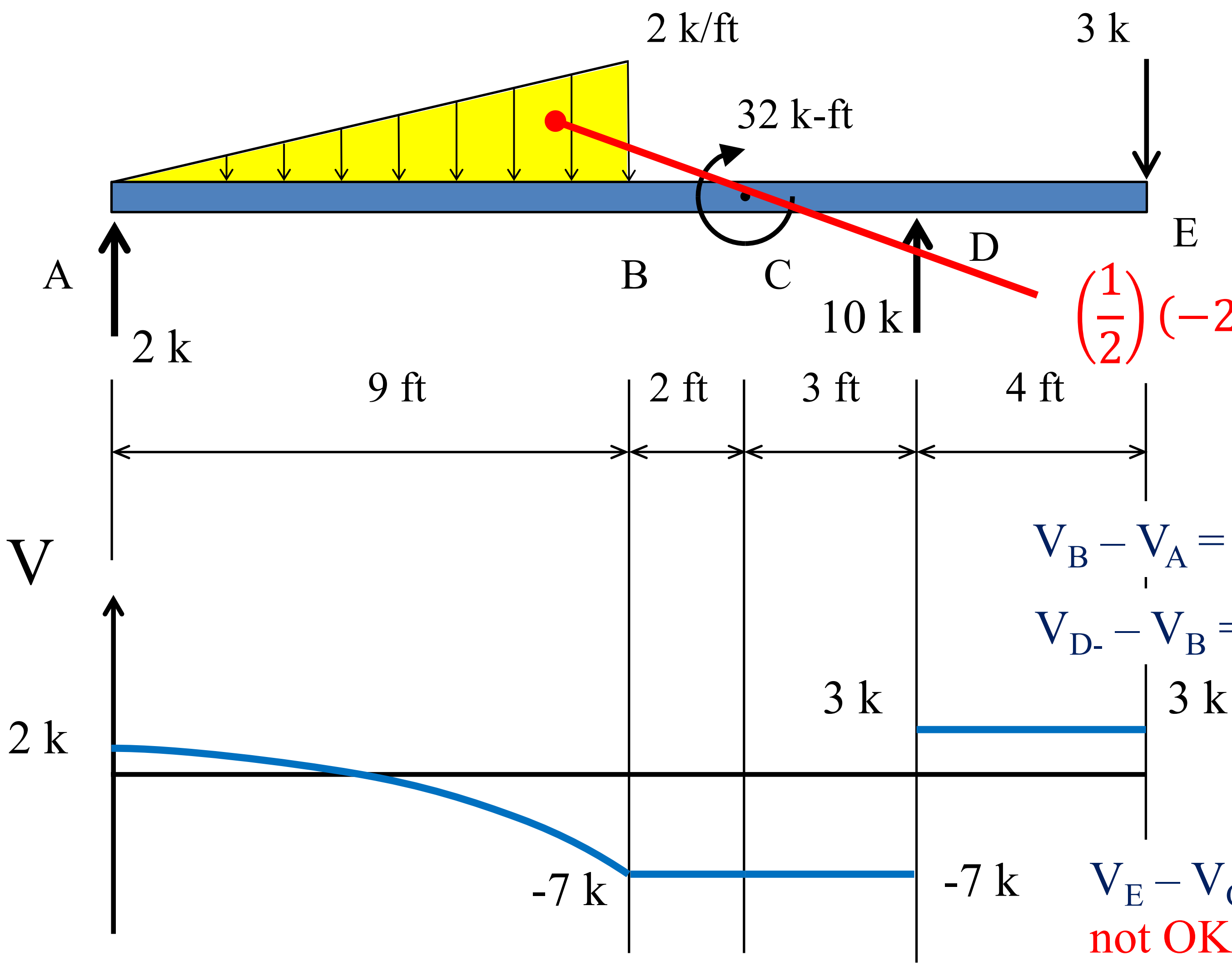


Slope of tangent line to the shear diagram at a point = Value of the magnitude of the distributed load intensity at that point

$$\frac{dV}{dx} = w$$



$$V_B - V_A = \int_A^B w\,dx = \text{area under the distributed load between points A and B}$$



$\left(\frac{1}{2}\right)(-2\ \text{k/ft})(9\ \text{ft}) = \mathbf{-9\ k}$

$V_B - V_A = -7 - 2 = \mathbf{-9\ k}$ OK

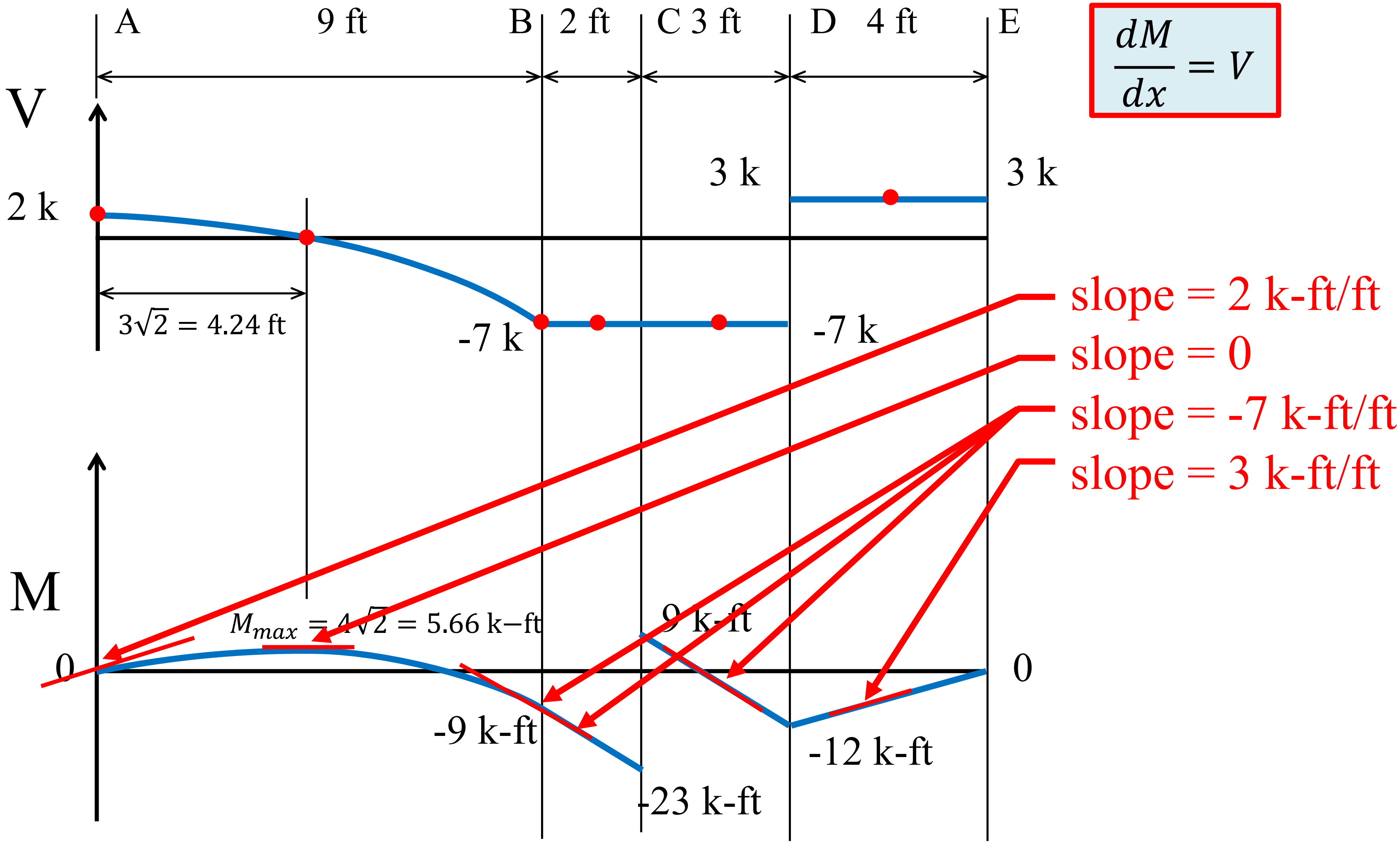
$V_{D-} - V_B = -7 - 7 = \mathbf{0}$ OK

$V_E - V_C = 3 - (-7) = \mathbf{10\ k}$

not OK why?

Slope of tangent line to the moment diagram at a point = Value of the magnitude of the ordinate of the shear diagram at that point

$$\frac{dM}{dx} = V$$



$$M_B - M_A = \int_A^B V\,dx = $$ area under the shear diagram between points A and B