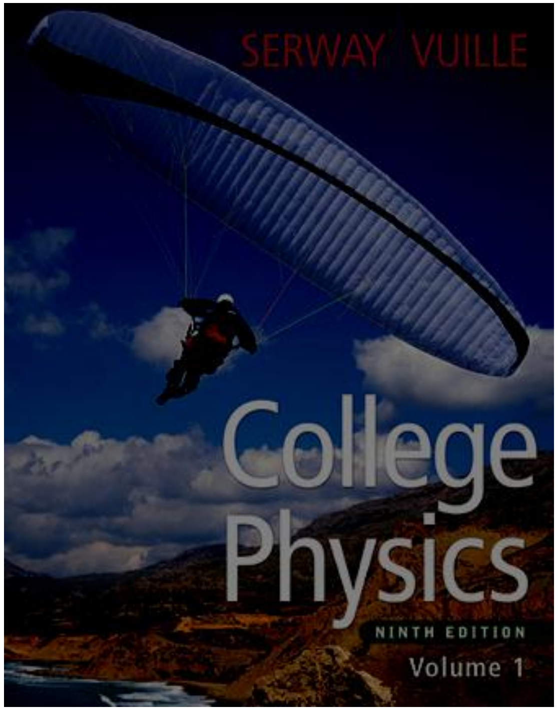

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CENGAGE Learning

Chapter 18

Direct Current Circuits

Quiz 4

1. An electron initially at rest accelerates through a potential difference of 1 V, gaining kinetic energy *KEe*, whereas a proton, also initially at rest, accelerates through a potential difference of -1 V, gaining kinetic ergy *KEp*. Which of the following relationships holds? (a) *KEe* = *KEp* (b) *KEe* < *KEp* (c) *Ke* > *KEp* (d) The answer can’t be determined from the given information. ne

2. A capacitor is designed so that one plate is large and the other is small. If the plates are connected to a battery, (a) the large plate has a greater charge than the small plate, (b) the large plate has less charge than the small plate, or (c) the plates have equal, but opposite, charge

Electric Circuits

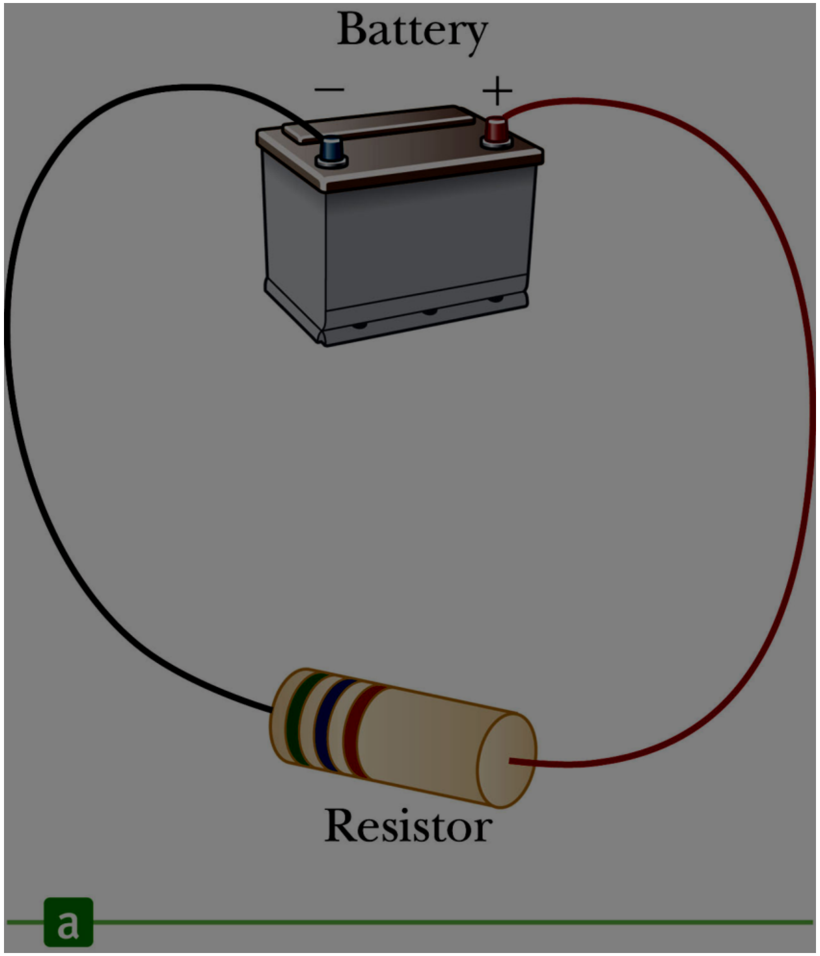
- Electric circuits control the flow of electricity and the energy associated with it.
- Circuits are used in many applications.
- Kirchhoff's Rules will simplify the analysis of simple circuits.
- Some circuits will be in steady state.
  - The currents are constant in magnitude and direction.
- In circuits containing resistors and capacitors, the current varies in time.

Sources of emf

- The source that maintains the current in a closed circuit is called a source of *emf.*
  - Any devices that increase the potential energy of charges circulating in circuits are sources of emf.
  - Examples include batteries and generators
- SI units are Volts
  - The emf is the work done per unit charge.

emf and Internal Resistance

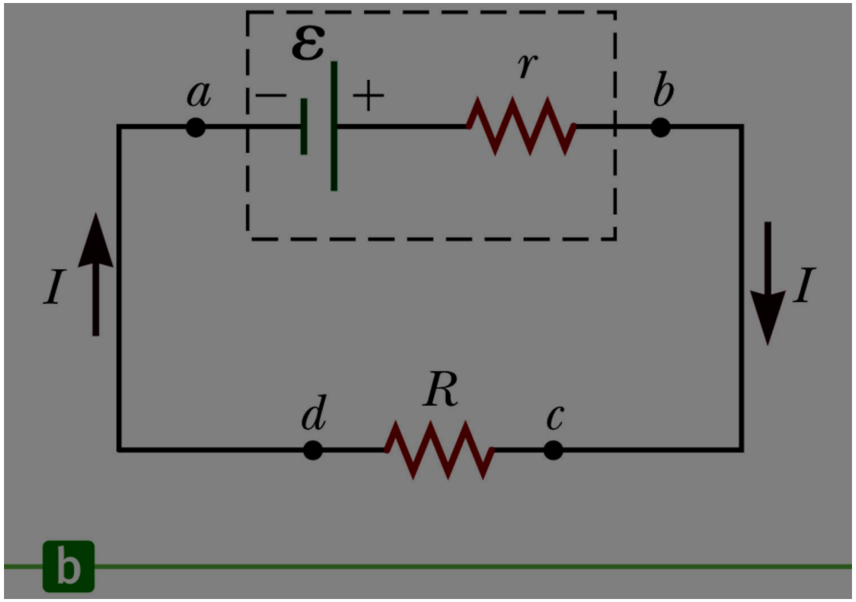
- A real battery has some internal resistance.
- Therefore, the terminal voltage is not equal to the emf.



More About Internal Resistance

- The schematic shows the internal resistance, r
- The terminal voltage is $\Delta V = V_b - V_a$
- $\Delta V = \varepsilon - Ir$
- For the entire circuit,

  $\varepsilon = IR + Ir$



Internal Resistance and emf, Cont.

- ε is equal to the terminal voltage when the current is zero.
  - Also called the *open-circuit voltage*
- R is called the *load resistance.*
- The current depends on both the resistance external to the battery and the internal resistance.

Internal Resistance and emf, Final

- When R >> r, r can be ignored.
  - Generally assumed in problems
- Power relationship
  - I ⍰ = $I^2 R + I^2 r$
- When R >> r, most of the power delivered by the battery is transferred to the load resistor.

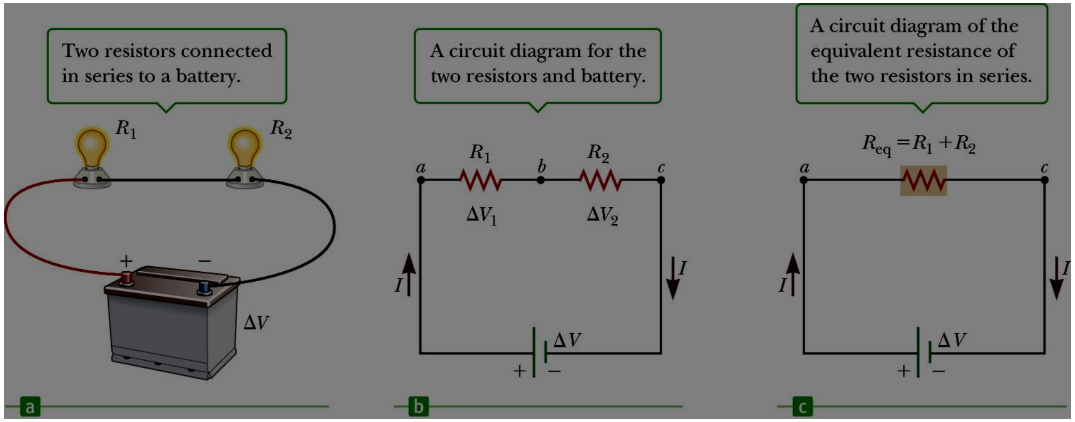
Batteries and emf

- The current in a circuit depends on the resistance of the battery.
  - The battery cannot be considered a source of constant current.
- The terminal voltage of battery cannot be considered constant since the internal resistance may change.
- The battery *is* a source of constant emf.

Resistors in Series

- When two or more resistors are connected end-to-end, they are said to be in *series.*
- The current is the same in all resistors because any charge that flows through one resistor flows through the other.
- The sum of the potential differences across the resistors is equal to the total potential difference across the combination.

Resistors in Series, Cont.

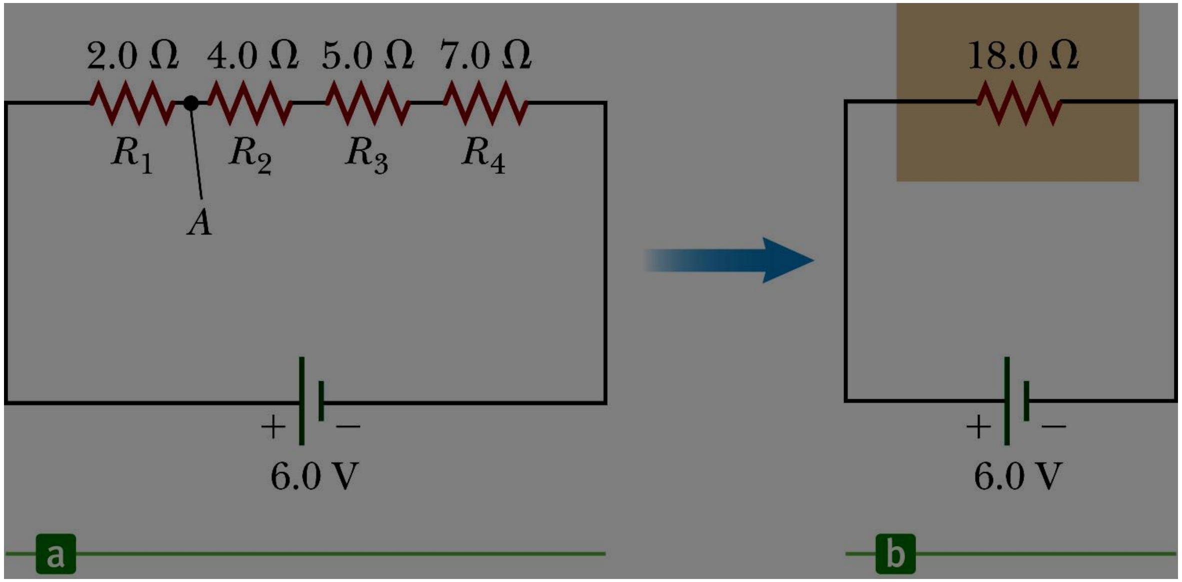


- Potentials add
  - $\Delta V = IR_1 + IR_2 = I\,(R_1+R_2)$
  - Consequence of Conservation of Energy
- The equivalent resistance has the effect on the circuit as the original combination of resistors.

Equivalent Resistance – Series

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$
- The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any of the individual resistors.
- If one element in the series circuit fails, the circuit would no longer be complete and none of the elements would work.

Equivalent Resistance – Series
An Example

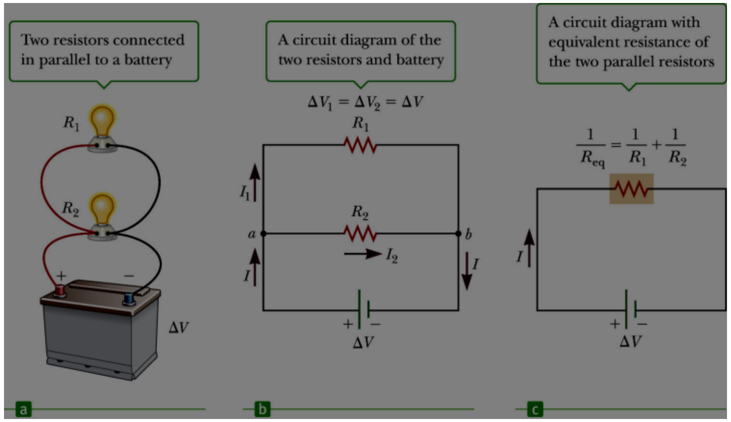


- Four resistors are replaced with their equivalent resistance.

Resistors in Parallel

- The potential difference across each resistor is the same because each is connected directly across the battery terminals.
- The current, I, that enters a point must be equal to the total current leaving that point.
  - $I = I_1 + I_2$
  - The currents are generally not the same.
  - Consequence of Conservation of Charge

Equivalent Resistance – Parallel
An Example



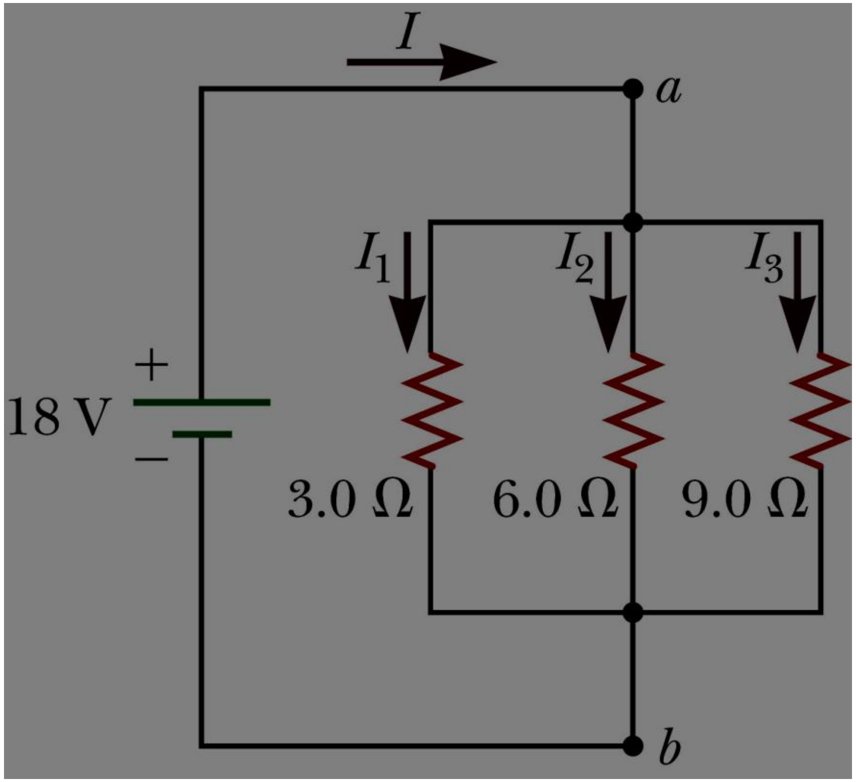
- Equivalent resistance replaces the two original resistances.
- *Household circuits* are wired so the electrical devices are connected in parallel.
  – Circuit breakers may be used in series with other circuit elements for safety purposes.

Equivalent Resistance – Parallel

- Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.
  - The equivalent is always less than the smallest resistor in the group.



Problem-Solving Strategy, 1

- Combine all resistors in series.
  - They carry the same current.
  - The potential differences across them are not the same.
  - The resistors add directly to give the equivalent resistance of the series combination: $R_{eq} = R_1 + R_2 + \ldots$
  - Draw the simplified circuit diagram.

Problem-Solving Strategy, 2

- Combine all resistors in parallel.
  - The potential differences across them are the same.
  - The currents through them are not the same.
  - The equivalent resistance of a parallel combination is found through reciprocal addition:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

  - Remember to invert the answer after summing the reciprocals.
  - Draw the simplified circuit diagram.

Problem-Solving Strategy, 3

- Repeat the first two steps as necessary.
  - A complicated circuit consisting of several resistors and batteries can often be reduced to a simple circuit with only one resistor.
  - Replace any resistors in series or in parallel using steps 1 or 2.
  - Sketch the new circuit after these changes have been made.
  - Continue to replace any series or parallel combinations.
  - Continue until one equivalent resistance is found.

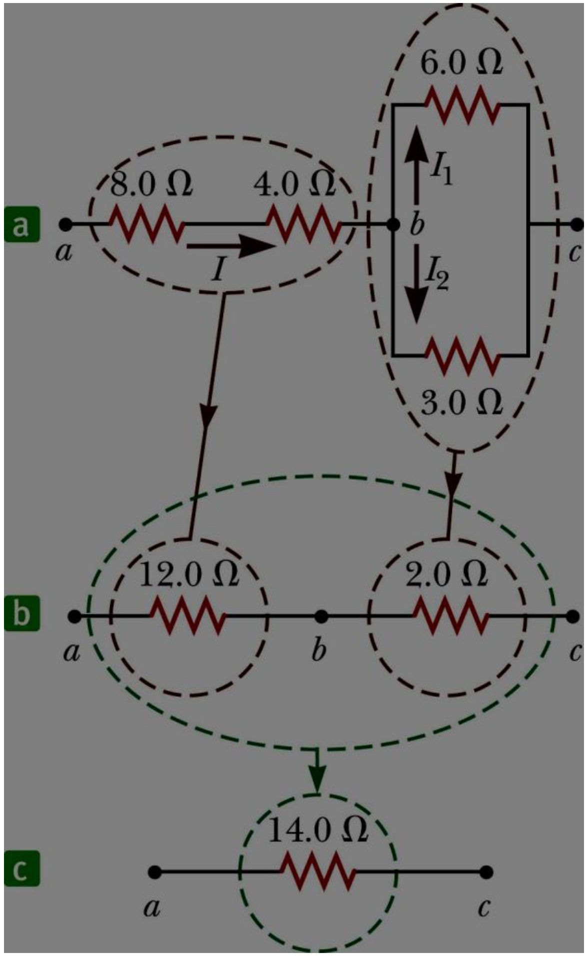
Problem-Solving Strategy, 4

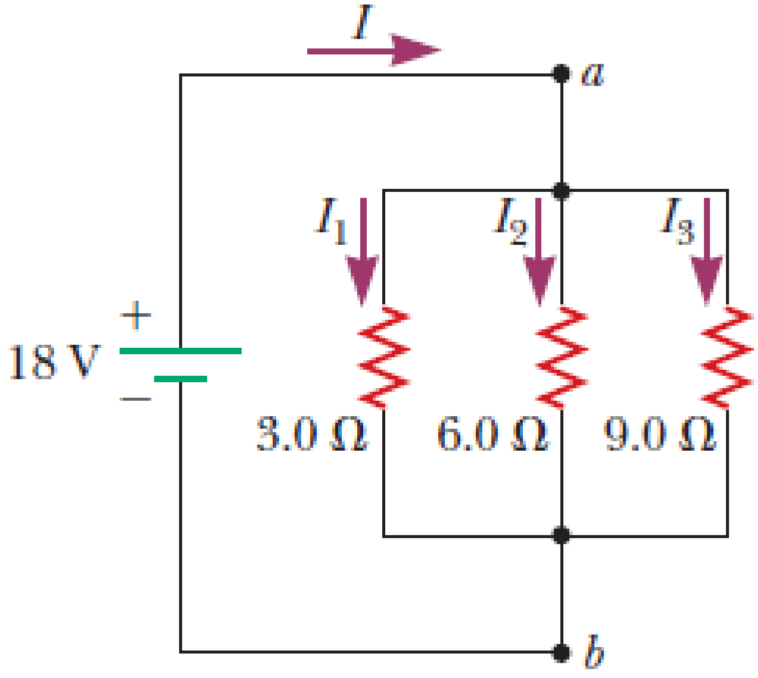
- Use Ohm's Law.
  - Use $\Delta V = I R$ to determine the current in the equivalent resistor.
  - Start with the final circuit found in step 3 and gradually work back through the circuits, applying the useful facts from steps 1 and 2 to find the current in the other resistors.

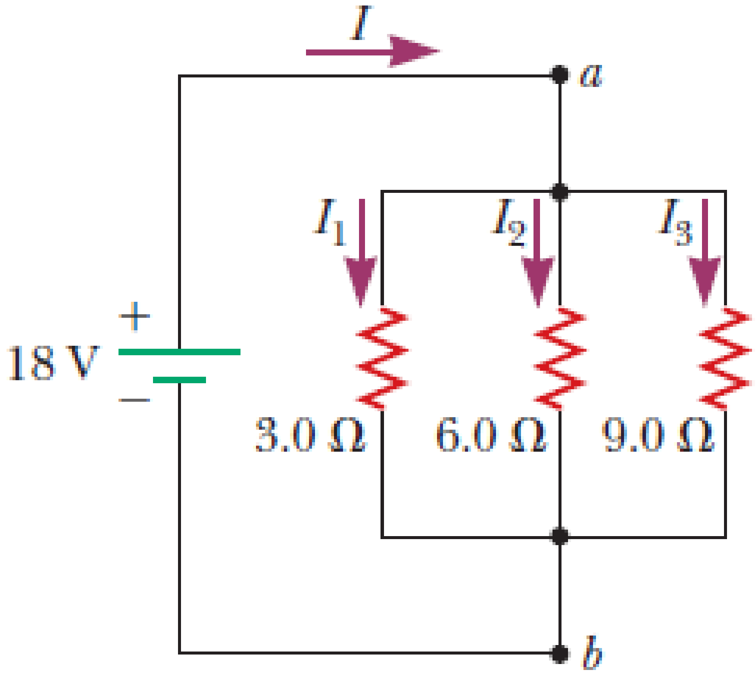
Three resistors are connected in parallel as in Figure. A potential difference of 18 V is maintained between points *a* and *b.* **(a)** Find the current in each resistor. **(b)** Calculate the power delivered to each resistor and the total power. **(c)** Find the equivalent resistance of the circuit. **(d)** Find the total power delivered to the equivalent resistance.

Example

- Complex circuit reduction
  - –Combine the resistors in series and parallel.
- Redraw the circuit with the equivalents of each set.
  - –Combine the resulting resistors in series.
  - –Determine the final equivalent resistance.




I
a
I_1
I_2
I_3
+
18 V
−
3.0 Ω
6.0 Ω
9.0 Ω
b



$$I_1 = \frac{\Delta V}{R_1} = \frac{18\text{ V}}{3.0\ \Omega} = \boxed{6.0\text{ A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18\text{ V}}{6.0\ \Omega} = \boxed{3.0\text{ A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18\text{ V}}{9.0\ \Omega} = \boxed{2.0\text{ A}}$$

$$3\ \Omega\text{: } P_1 = I_1^{\,2}R_1 = (6.0\text{ A})^2(3.0\ \Omega) = \boxed{110\text{ W}}$$

$$6\ \Omega\text{: } P_2 = I_2^{\,2}R_2 = (3.0\text{ A})^2(6.0\ \Omega) = \boxed{54\text{ W}}$$

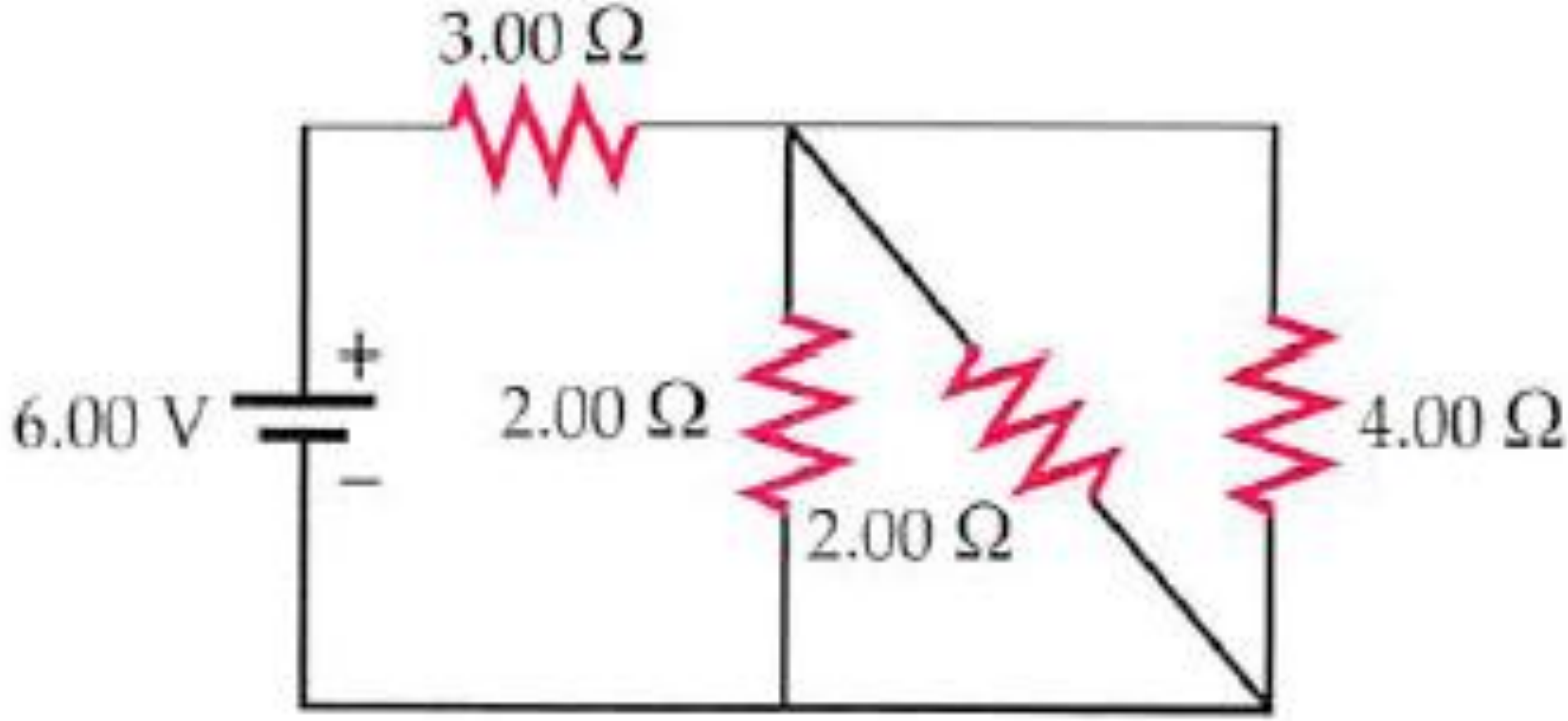
$$9\ \Omega\text{: } P_3 = I_3^{\,2}R_3 = (2.0\text{ A})^2(9.0\ \Omega) = \boxed{36\text{ W}}$$

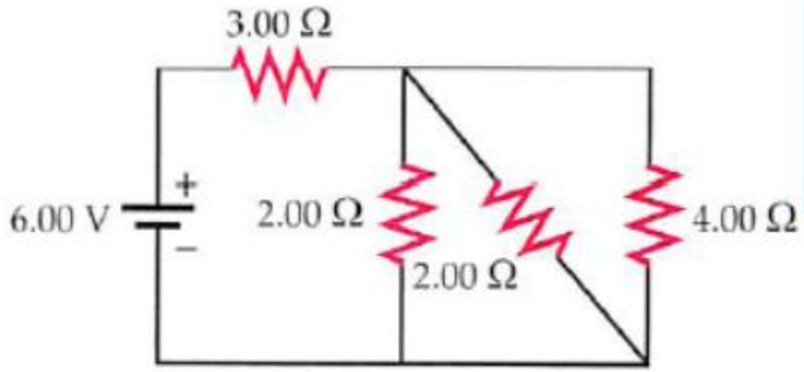
$$P_{\text{tot}} = 110\text{ W} + 54\text{ W} + 36\text{ W} = \boxed{2.0 \times 10^2\text{ W}}$$

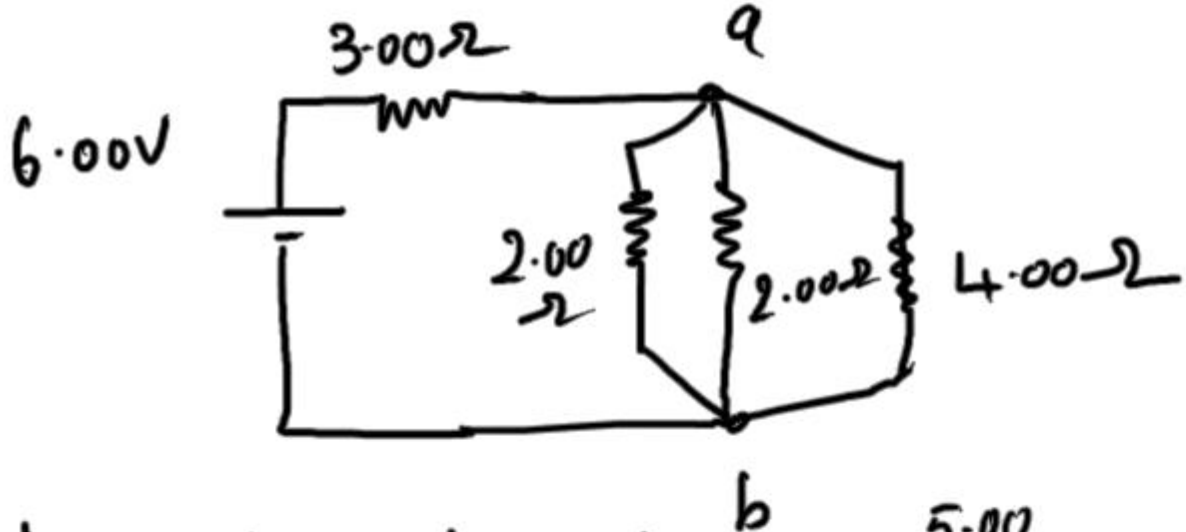
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{3.0\ \Omega} + \frac{1}{6.0\ \Omega} + \frac{1}{9.0\ \Omega} = \frac{11}{18\ \Omega}$$

$$R_{eq} = \frac{18}{11}\ \Omega = \boxed{1.6\ \Omega}$$

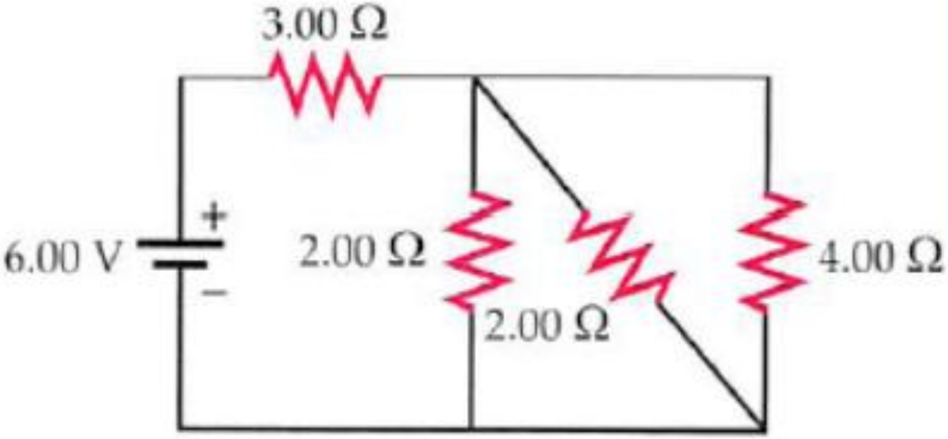

3.00 Ω
6.00 V
+
−
2.00 Ω
2.00 Ω
4.00 Ω

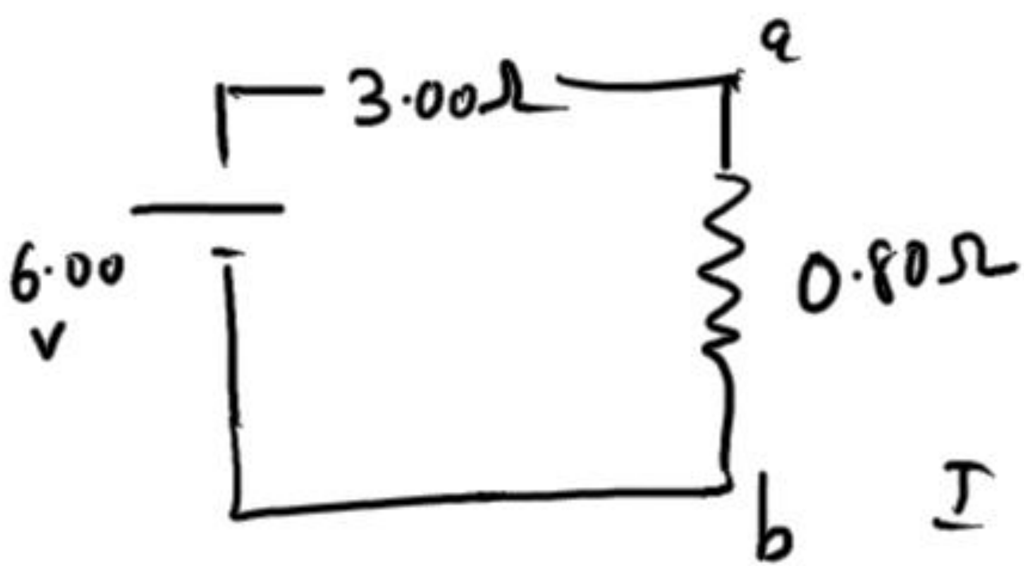




$$\frac{1}{R_{ab}} = \frac{1}{2.00\,\Omega} + \frac{1}{2.00\,\Omega} + \frac{1}{4.00\,\Omega} = \frac{5.00}{4.00\,\Omega}$$

$$R_{ab} = \frac{4.00}{5.00}\,\Omega = 0.80\,\Omega$$





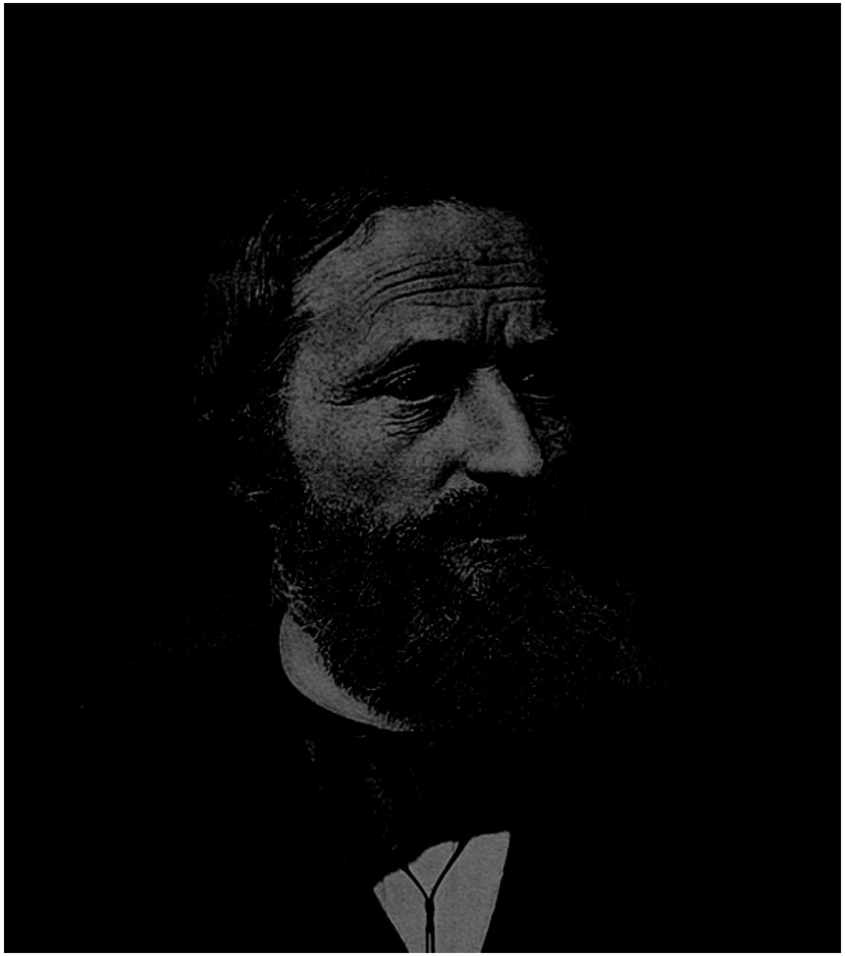
$$R_{Total} = 3.00\,\Omega + 0.800\,\Omega$$

$$= 3.80\,\Omega$$

$$I = \frac{6.00\,V}{3.80\,\Omega}\ \text{Amp}.$$

Gustav Kirchhoff

- 1824 – 1887
- Invented spectroscopy with Robert Bunsen
- Formulated rules about radiation



Kirchhoff’s Rules

- There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.
- Two rules, called Kirchhoff’s Rules, can be used instead.

Statement of Kirchhoff’s Rules

- Junction Rule

– The sum of the currents entering any junction must equal the sum of the currents leaving that junction.

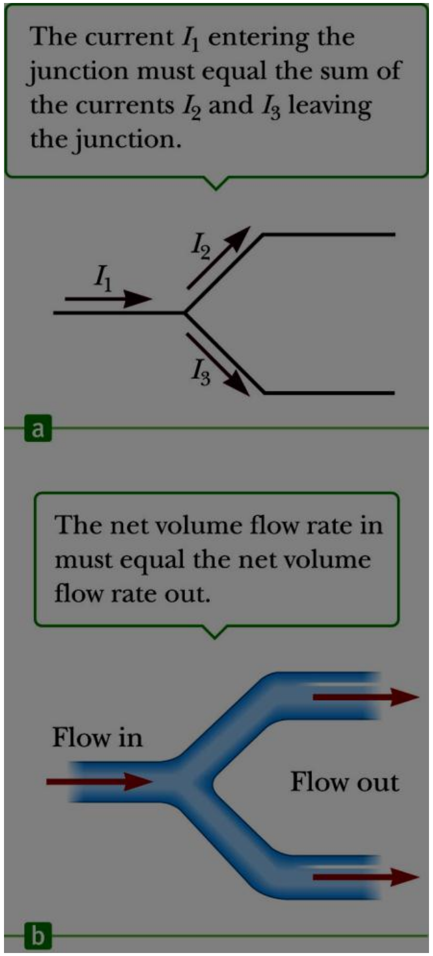
- A statement of Conservation of Charge
- Loop Rule

– The sum of the potential differences across all the elements around any closed circuit loop must be zero.

- A statement of Conservation of Energy

More About the Junction Rule

- $I_1 = I_2 + I_3$
- From Conservation of Charge
- Diagram b shows a mechanical analog.



Loop Rule

- A statement of Conservation of Energy
- To apply Kirchhoff's Rules,

–Assign symbols and directions to the currents in all branches of the circuit.

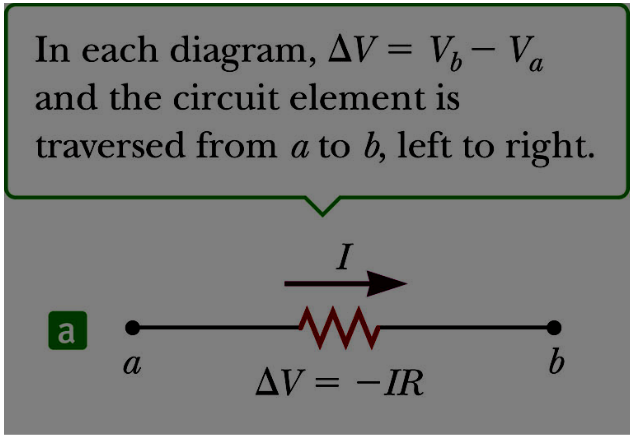
- If the direction of a current is incorrect, the answer will be negative, but have the correct magnitude.

–Choose a direction to transverse the loops.

- Record voltage rises and drops.

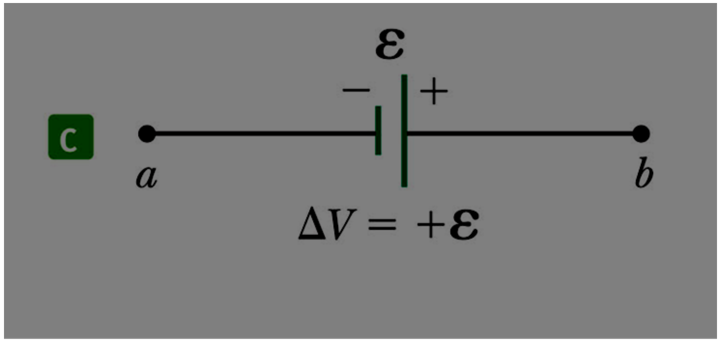
More About the Loop Rule

- Traveling around the loop from a to b
- In a, the resistor is transversed in the direction of the current, the potential across the resistor is –IR.
- In b, the resistor is transversed in the direction opposite of the current, the potential across the resistor is +IR.



Loop Rule, Final

- In c, the source of emf is transversed in the direction of the emf (from – to +), the change in the electric potential is +ε
- In d, the source of emf is transversed in the direction opposite of the emf (from + to -), the change in the electric potential is -ε

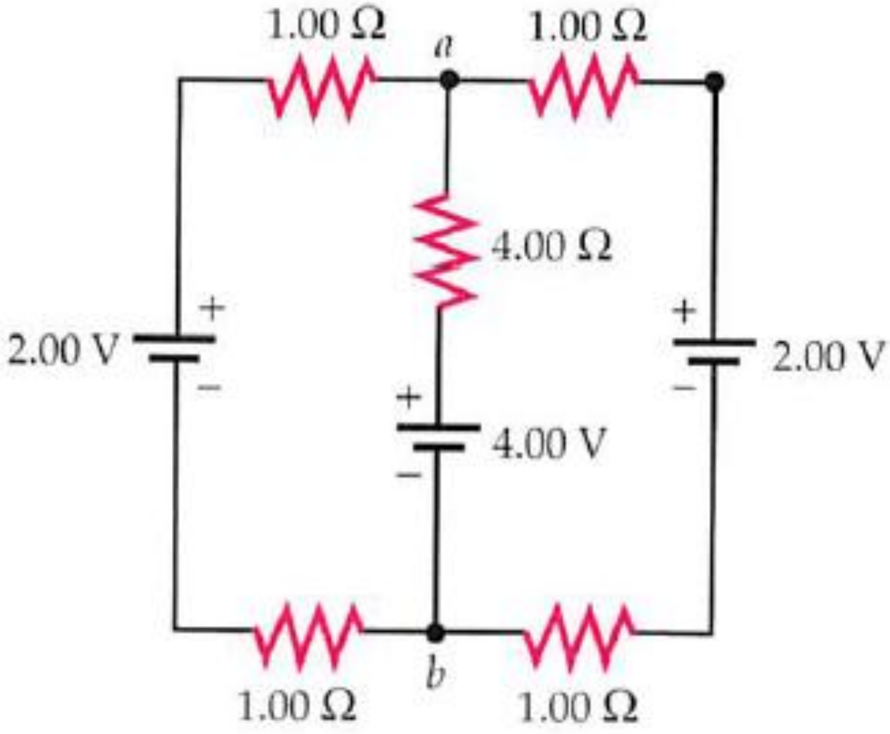


Junction Equations from Kirchhoff's Rules

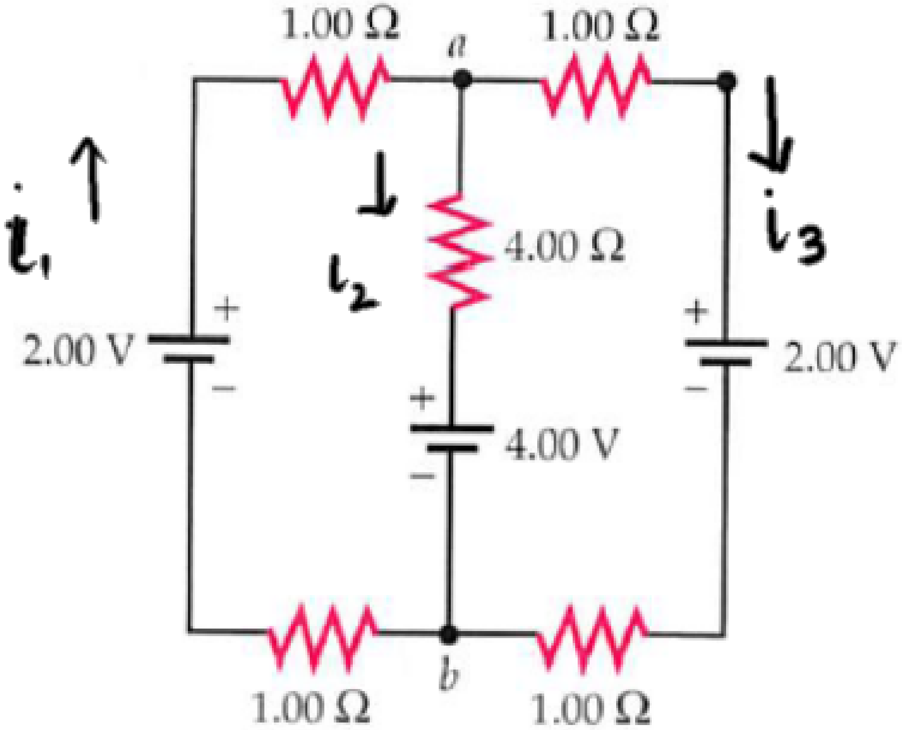
- Use the junction rule as often as needed, so long as, each time you write an equation, you include in it a current that has not been used in a previous junction rule equation.
  - In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.

Loop Equations from Kirchhoff's Rules

- The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation.
- You need as many independent equations as you have unknowns.



1.00 Ω
a
1.00 Ω
4.00 Ω
+
2.00 V
−
+
4.00 V
−
+
2.00 V
−
1.00 Ω
b
1.00 Ω



$i_1 = i_2 + i_3$ (a)

$i_2 + i_3 = i_1$ (b)

> same

$$2.00V - 1.00 \times i_1 V - 4.00 \times i_2 V - 4.00V - 1.00 i_1 = 0$$

$$-2.00 i_1 - 4.00 i_2 - 2.00V = 0 \text{ or}$$

$$2.00 i_1 + 4.00 i_2 = -2.00 \quad (1)$$

$$2.00V - 1.00 i_1 V - 1.00 i_3 V - 2.00V - 1.00 i_3 V - 1.00 i_1 V = 0$$

$$+2.00 i_1 + 2.00 i_3 = 0$$

$$i_1 = i_2 + i_3 \quad (1)$$

$$2.00 i_1 + 4.00 i_2 = -2.00 \quad (2)$$

$$2.00 i_1 + 2.00 i_3 = 0 \quad (3)$$

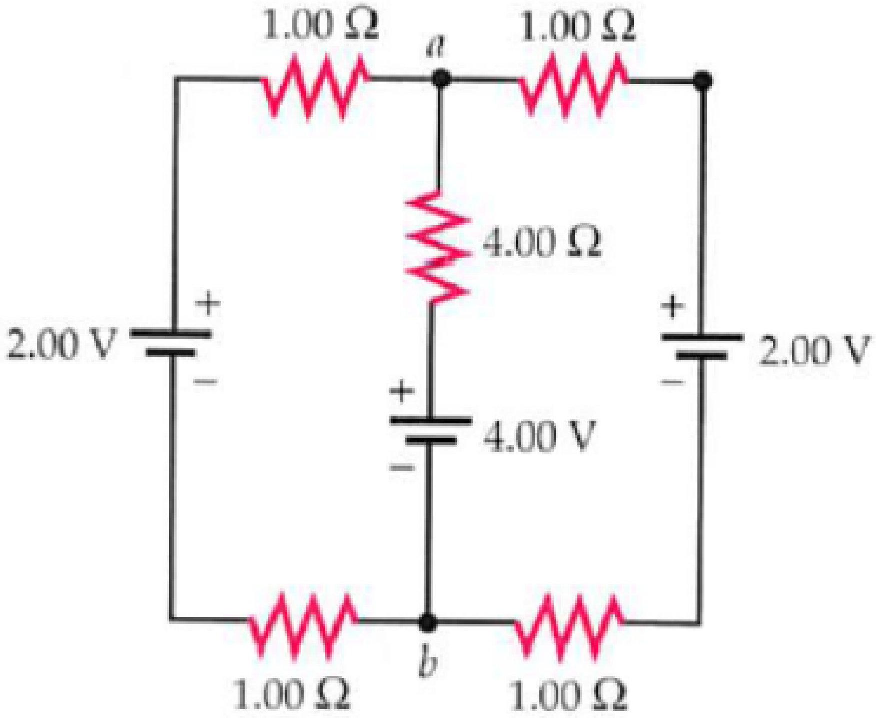
$$\therefore \boxed{i_1 = -i_3}$$

$$-i_3 = i_2 + i_3; \quad -2i_3 = i_2 \quad (4) \quad \boxed{i_3 = -0.5\, i_2}$$

$$-2.00 i_3 + 4.00(-2i_3) = -2.00$$

$$-10.00 i_3 = -2.00; \quad i_3 = 0.2$$

$$i_3 = 0.2\,A; \quad i_1 = -0.2\,A \quad i_2 = -0.4\,A$$



$$2.00 - (1.00)(-0.2) - (4.00)(-0.4) - 4.00$$

① $- (1.00)(-0.2)$

$$= 2.00 + 0.2 + 1.6 - 4.0 + 0.2 = 0!$$

② $2.00 + 0.2 - 0.2 - 2.00 - 0.2 + 0.2$

$$= 0!$$

Problem-Solving Strategy – Kirchhoff's Rules

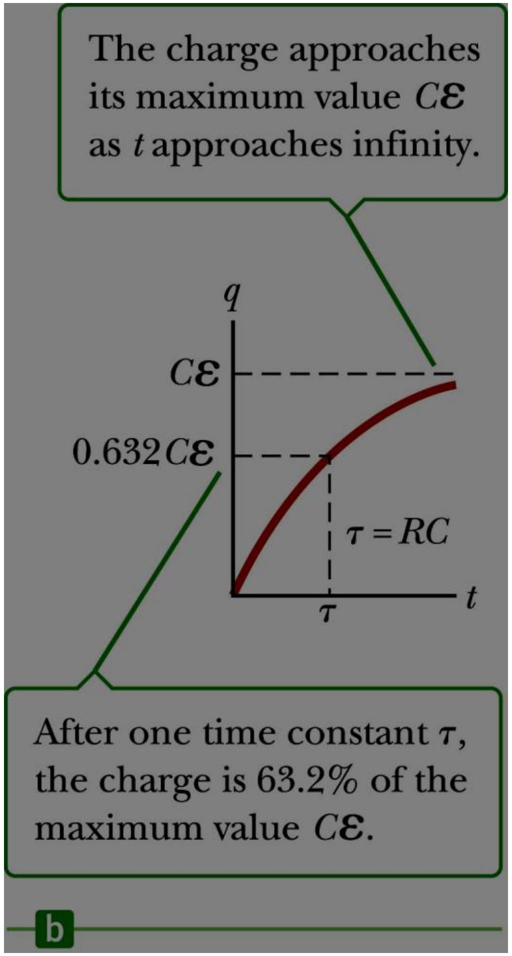
- Draw the circuit diagram and assign labels and symbols to all known and unknown quantities.
- Assign directions to the currents.
- Apply the junction rule to any junction in the circuit.
- Apply the loop rule to as many loops as are needed to solve for the unknowns.
- Solve the equations simultaneously for the unknown quantities.
- Check your answers.

RC Circuits

- When a direct current circuit contains capacitors and resistors, the current will vary with time.
- When the circuit is completed, the capacitor starts to charge.
- The capacitor continues to charge until it reaches its maximum charge ($Q = C\varepsilon$).
- Once the capacitor is fully charged, the current in the circuit is zero.

Charging Capacitor in an RC Circuit

- The charge on the capacitor varies with time.
  - $q = Q(1 - e^{-t/RC})$
  - The *time constant*, ?=RC
- The time constant represents the time required for the charge to increase from zero to 63.2% of its maximum.

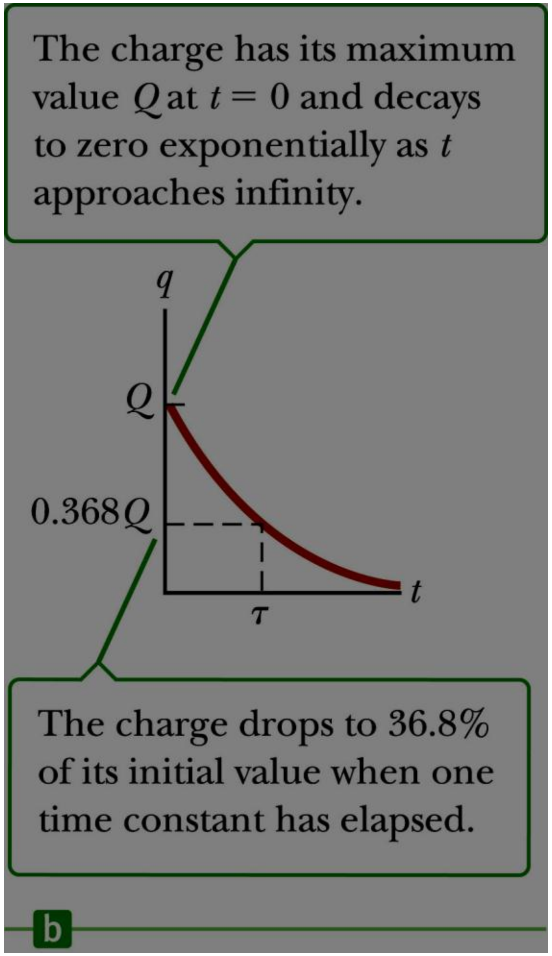


Notes on Time Constant

- In a circuit with a large time constant, the capacitor charges very slowly.
- The capacitor charges very quickly if there is a small time constant.
- After t = 10 ⍰, the capacitor is over 99.99% charged.

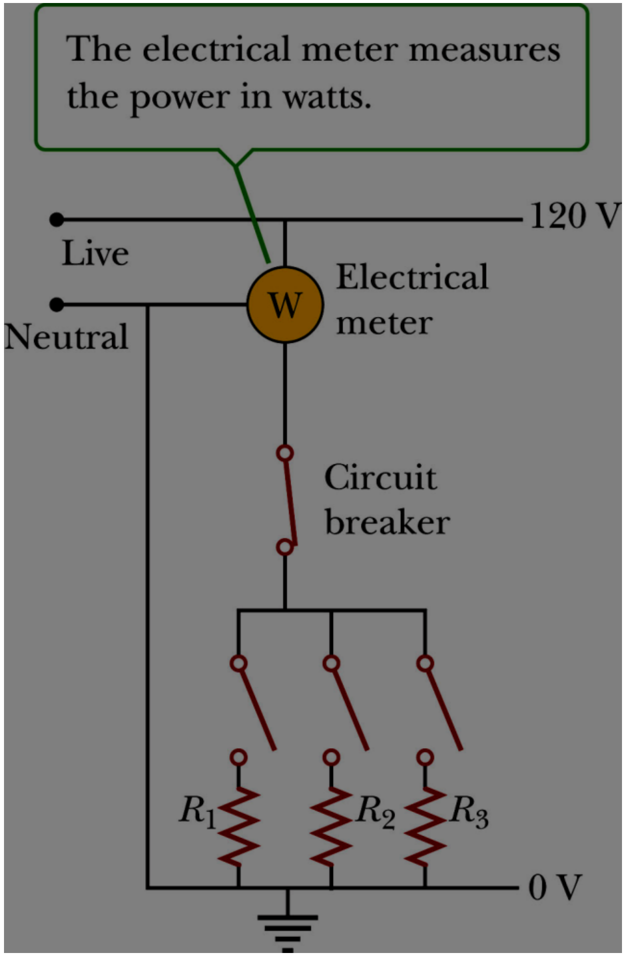
Discharging Capacitor in an RC Circuit

- When a charged capacitor is placed in the circuit, it can be discharged.
  - $q = Qe^{-t/RC}$
- The charge decreases exponentially.
- At t = ⍰ = RC, the charge decreases to 0.368 Q_{max}
  - In other words, in one time constant, the capacitor loses 63.2% of its initial charge.



Household Circuits

- The utility company distributes electric power to individual houses with a pair of wires.
- Electrical devices in the house are connected in parallel with those wires.
- The potential difference between the wires is about 120V.

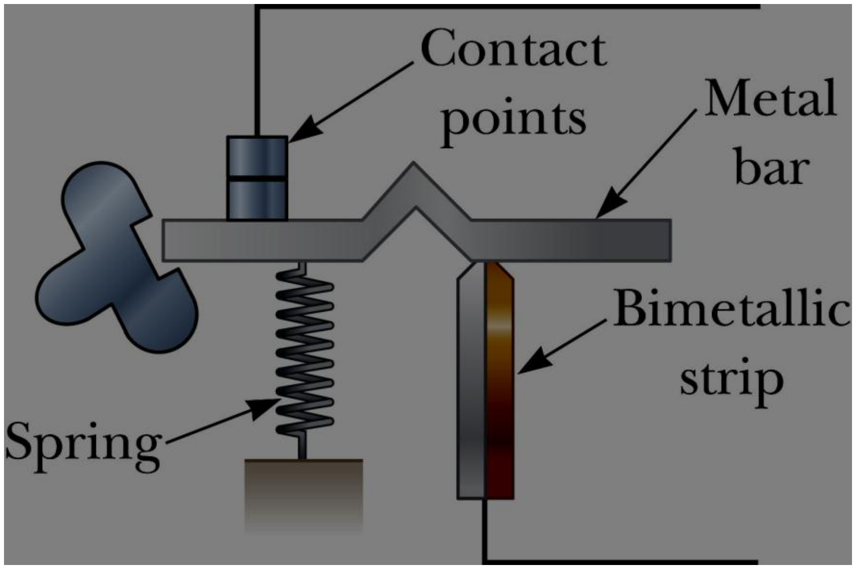


Household Circuits, Cont.

- A meter and a circuit breaker are connected in series with the wire entering the house.
- Wires and circuit breakers are selected to meet the demands of the circuit.
- If the current exceeds the rating of the circuit breaker, the breaker acts as a switch and opens the circuit.
- Household circuits actually use alternating current and voltage.

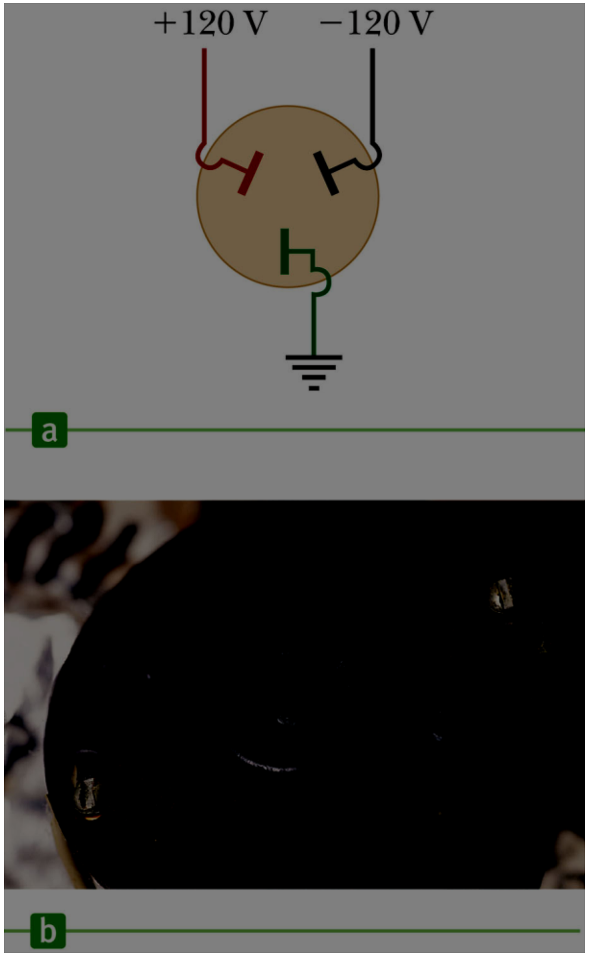
Circuit Breaker Details

- Current passes through a bimetallic strip.
  - The top bends to the left when excessive current heats it.
  - Bar drops enough to open the circuit
- Many circuit breakers use electromagnets instead.



240-V Connections

- Heavy-duty appliances may require 240 V to operate.
- The power company provides another wire at 120 V below ground potential.



Electrical Safety

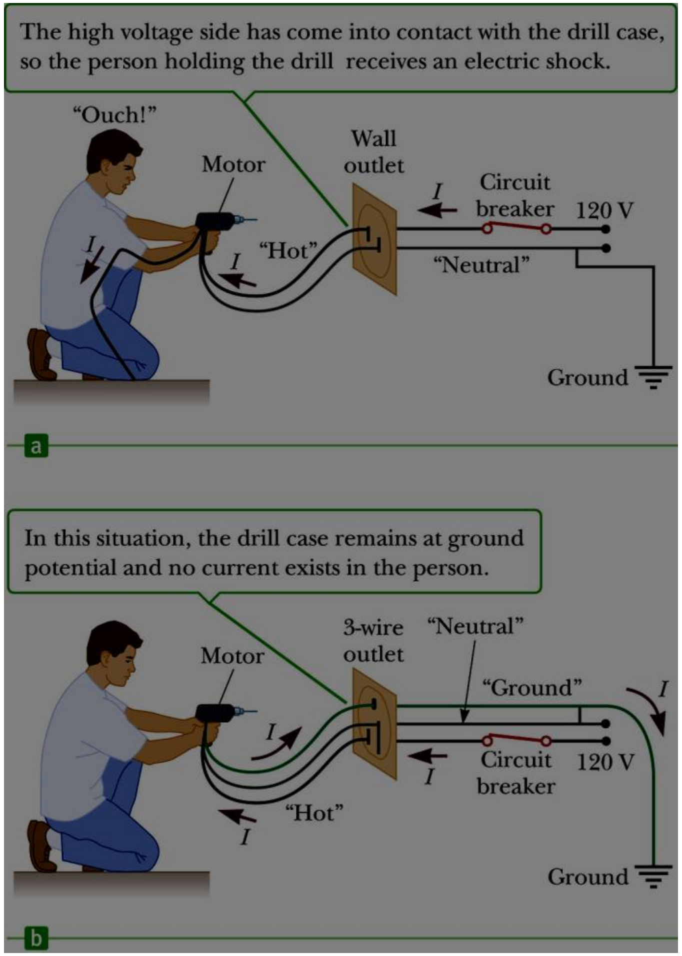
- Electric shock can result in fatal burns.
- Electric shock can cause the muscles of vital organs (such as the heart) to malfunction.
- The degree of damage depends on
  - The magnitude of the current
  - The length of time it acts
  - The part of the body through which it passes

Effects of Various Currents

- 5 mA or less
  - Can cause a sensation of shock
  - Generally little or no damage
- 10 mA
  - Hand muscles contract
  - May be unable to let go of a live wire
- 100 mA
  - If passes through the body for just a few seconds, can be fatal

Ground Wire

- Electrical equipment manufacturers use electrical cords that have a third wire, called a *case ground.*
- Prevents shocks



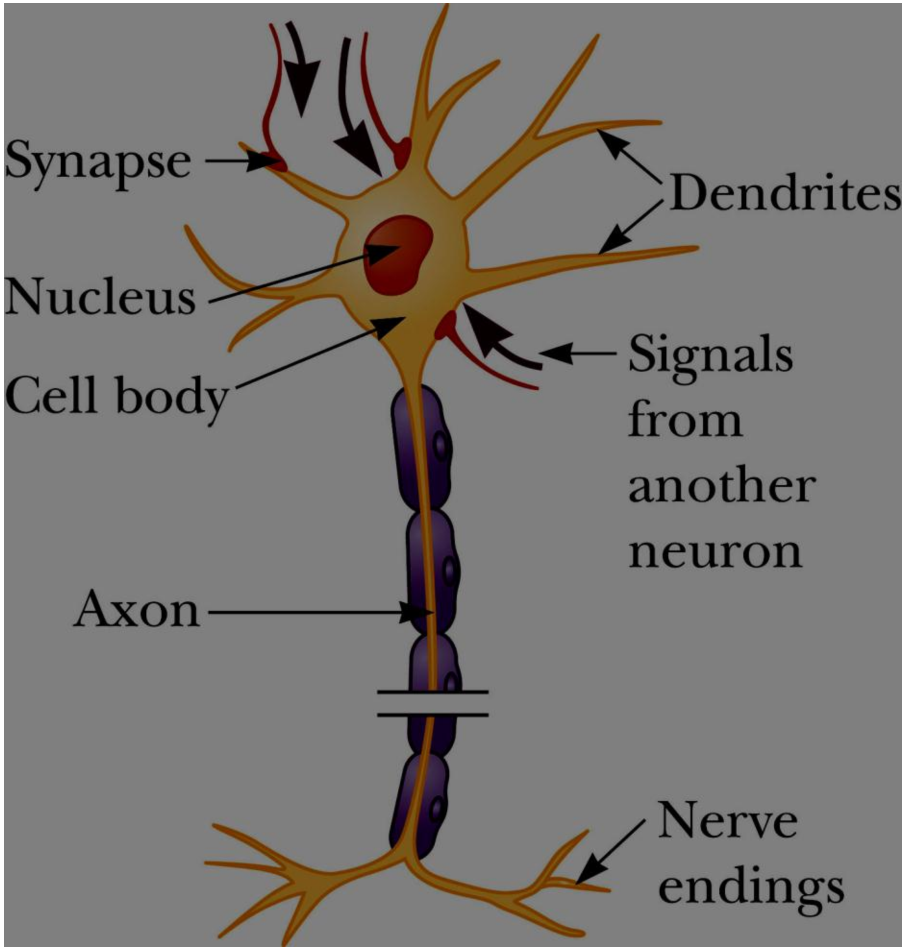
Ground Fault Interrupts (GFI)

- Special power outlets
- Used in hazardous areas
- Designed to protect people from electrical shock
- Senses currents (of about 5 mA or greater) leaking to ground
- Shuts off the current when above this level

Electrical Signals in Neurons

- Specialized cells in the body, called *neurons*, form a complex network that receives, processes, and transmits information from one part of the body to another.
- Three classes of neurons
  - Sensory neurons
    - Receive stimuli from sensory organs that monitor the external and internal environment of the body
  - Motor neurons
    - Carry messages that control the muscle cells
  - Interneurons
    - Transmit information from one neuron to another

Diagram of a Neuron



Section 18.8